# AUSTRALIAN MARITIME COLLEGE

SEMESTER 2 - FINAL EXAM EXAMINATION 2003

SOLUTIONS TO EXAMINATION PAPER

SUBJECT INSTR & PROCESS CONTROL TIME ALLOWED 3 HOURS

NO OF QUESTIONS TO BE ATTEMPED SIX (6)

| QUESTION NO | SOLUTION | MARK ANALYSIS |
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| **01** | (a) According to the reference sources, there is no exact definition for instrumentation. The concept of instrumentation is as follows: in general, instrumentation is the application of instruments for monitoring, sensing, and measurement. The purposes of instrumentation may be, not restricted to:<br><br>• product testing and quality control<br>• monitoring in the interest of health, safety, or costing;<br>• part of a control system, or<br>• research and development<br><br>The key to successful instrumentation is undoubtedly accurate measurement, which can be achieved only if the needs and limitations of the instrumentation systems are understood. Measurement can be defined as a comparison with a standard, thus one should always be aware of the basic standard against which the comparison is being made. Calibration and periodic routine maintenance of the instruments are prerequisites of accurate measurement, without which one may too often waste time and effort recording meaningless readings and results.<br><br>The reason why instrumentation is important in a control system is that in a control system, one or many quantities need to be controlled and monitored. The instrumentation systems provide means to measure such quantities and then convert them into appropriate forms of signal that are necessary for a control system. The instrumentation systems (sensor, monitor and transmitter) are indispensable to an advanced control system.<br><br>(b) The general block diagram of a measuring system can be represented as follows (Figure 1). | 2<br><br>2 |

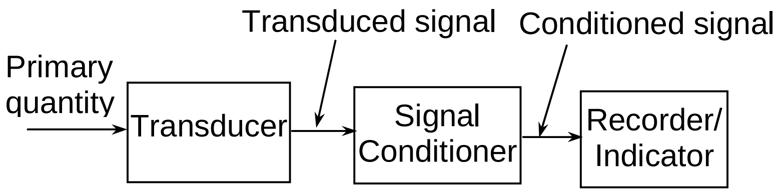


Figure 1 Block diagram of a basic measurement system

<u>Transducer Block:</u> has a function as an energy converter that receives the physical quantity being measured and converts it into some other physical variable; eg flow to pressure, speed to voltage, stain to resistance. The transducer is undoubtedly the weakest link in the measuring chain, for measured quantity is always modified by the presence of the transducer, making a perfect measurement theoretically impossible.

<u>Signal Conditioner Block:</u> has a function to rearrange the transduced signal into a form which can be readily recorded or monitored.

<u>Recorder Block:</u> may be a recorder, display, or indicating device. The recorder block has a function to record or indicate the measure quantity.

An example of the measuring device used in marine and offshore systems is the Gyrocompass that measures the ship's course. The gyrocompass can be expressed by the following block diagram:

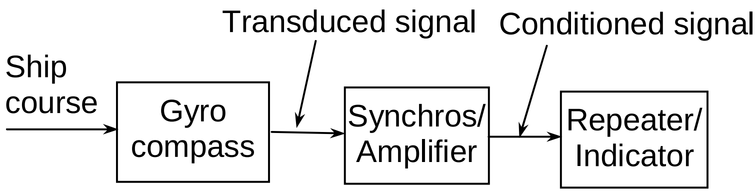


Figure 2 Block diagram of a gyrocompass

Gyrocompass is a transducer that converts the ship course signal into the electrical form.

The Synchros and Amplifier block as a conditioning devices rearranges the ship course signal into the form that can be readily indicated by the repeaters.

The Repeater block is a recorder/indicator that shows an operator the reading of the ship course.

| | | |
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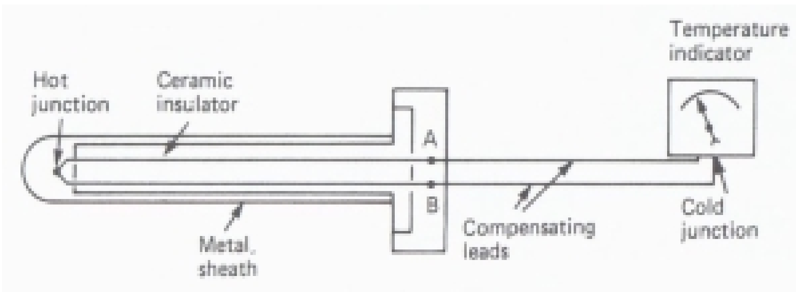
| | (c) Measuring systems can be classified as:<br>(i) Mechanical measuring system<br>(ii) Electrical & electronic measuring system<br>where electronic measuring systems are nowadays preferred because they have the very rapid speed of response and it is easy to increase the electrical and electronic signals in amplitude and transmit them over long distances.<br>Measuring can be classified according to their signal forms:<br>(i) Analogue measuring systems: that treat continuous time signals<br>(ii) Digital measuring systems: that treat discrete time signals<br>(iii) Hybrid measuring systems: which treat signals in both continuous time and discrete time forms, eg computer-based measuring systems.<br>The hybrid measuring systems are nowadays preferred because they can transmit data over long distances, or even through the Internet. | 2 |
| | (d) An example of a measuring method is taken as follows: temperature measuring system using a thermocouple. The description and principle of a thermocouple are given below. | 2 |
| | The thermocouple uses two different metals or alloys jointed together to make a closed circuit. When the two junctions are at different temperatures an e.m.f. (electromagnetic force) is generated and a current flows. The magnitude of the e.m.f. and the current flowing depend upon the temperature difference between the junctions. The arrangement used is shown in Figure 3, where extra wires or compensating leads are introduced to complete the circuit and include the indicator. As long as the two ends A and B are at the same temperature the thermoelectric effect is not influenced. The choice of metals will determine the measuring range, e.g. Copper-Constantan -200 to +350°C, Platinum/Platinum and Rhodium 0 to +1500°C.<br><br>Figure 3 Thermocouple | Sub: 8 |

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(a) An automatic control system, including it recording (indicating) elements, can be represented by the following general block diagram.

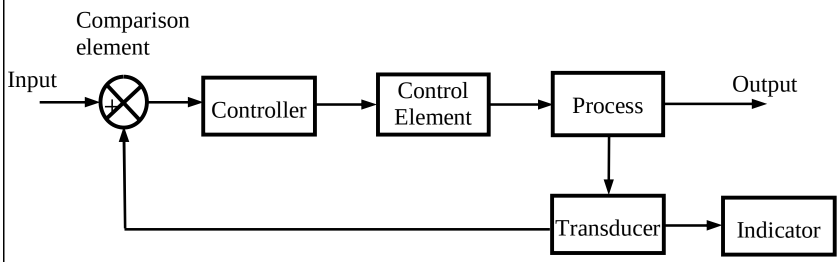


Figure 4 General block diagram of a control system

<u>Comparison Element Block:</u> compares the output or control variable with the desired input (reference signal) and generate an error or deviation signal to the controller. The comparison element performs the mathematical operation of subtraction.

<u>Controller Block:</u> calculates control signals from the error signal, and generates the control signals to the Control Element. The controller block can be a PID controller where the control signal.

<u>Control Element Block:</u> Control element block is the element in which the amplified and conditioned control signal is used to regulate some energy source to the process. The control element block is often referred as an actuator. Control element block may be a valve or a motor.

<u>Process Block</u>: is the dynamic system where the process is implemented.

Transducer Block: is a sensing device that receives the physical quantity being measured from the process, converts it into some other physical variable and generates this signal to an indicating device or feeds back to the comparison element.

<u>Indicator Block:</u> indicates or records the measured quantity.
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(b) (The answer may be a differential lever, potentiometer, or synchros): Synchros are the a.c. equivalent of potentiometers and are used in many a.c. electrical systems for data transmission and torque transmission for driving dials. They are also used to compare input and output rotations in a.c. electrical servo-systems and rotating hydraulic systems.
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To perform error detection, two synchros are used: one in the mode of a control transmitter, and the other as a control transformer, as shown in Figure 5.

The synchros have their stator coils equally spaced at $120^o$ intervals. An a.c. voltage (often 115V at 400Hz) is applied to the transmitter rotor, producing voltages in the stator coils (by transformer action) which uniquely define the angular position of the rotor. These voltages are transmitted to the stator coils of the transformer, producing a resultant magnetic field aligned in the same direction as the transmitter rotor.

The transformer rotor acts as a "search coil" in detecting the direction of its stator field. The maximum voltage is induced in the transformer rotor coil when the rotor axis is aligned with the field. Zero voltage is induced when the rotor axis is perpendicular. The "in-line" position of the input and output shafts therefore requires the transformer rotor coil to be at $90^o$ to the transmitter rotor coil.

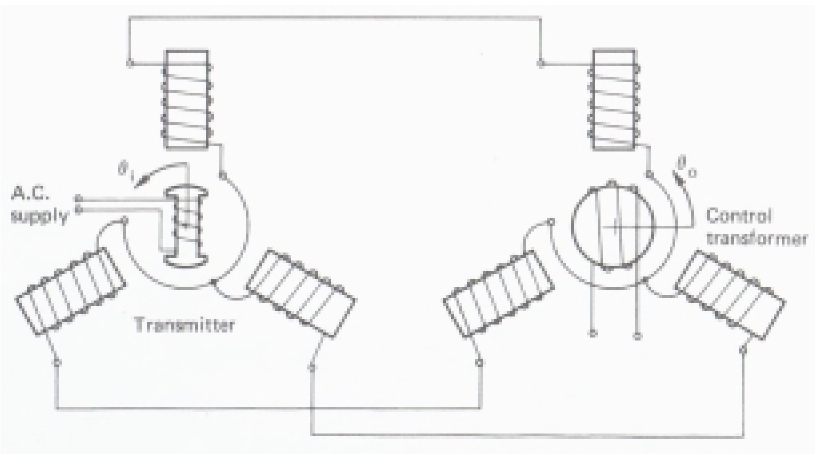


Figure 5 Error detection by synchros

The output voltage is an amplitude-modulated signal which requires demodulating to produce the following relationship for small misalignment angles:

Output = K (input-shaft position – output-shaft position)

$$= K(\theta_i - \theta_0)$$

where K = voltage gradient (volts/degree)

Compared to d.c. potentiometers, synchros have the following advantages:

(1) a full $360^o$ of shaft rotation is always available;

(2) since they have no sliding contacts, their life expectancy is much higher, resolution is infinite, and hence they do not have “noise” problems;
(3) a.c. amplifiers can be employed and therefore are no drift problems.

However, phase-sensitive rectifiers are necessary to sense direction.

<u>(c) A sensor or transducer used in the maritime industries</u>: 2

Transducers or sensors are the devices that convert the quantity being measured into an optical, mechanical, or more commonly – electrical signal. The energy-conversion process that takes place is referred to as transduction. Examples of transducers may be resistance displacement transducer, pressure bellows, force diaphragm, etc. The following is a brief description of pressure bellows.

The bellows is used in some pneumatic devices to provide feedback and also as a transducer to convert an input pressure signal into a displacement. A simple bellows arrangement is shown in Figure 6. The bellows will elongate when the supply pressure increases and some displacement, x, will occur. The displacement will be proportional to the force acting on the base, i.e. supply pressure $\times$ area. The actual amount of displacement will be determined by the spring-stiffness of the bellows. Thus

$$\begin{pmatrix}\text{Supply}\\ \text{pressure}\end{pmatrix}\times\begin{pmatrix}\text{Area of}\\ \text{bellows}\end{pmatrix}=\begin{pmatrix}\text{Spring}-\text{stiffness}\\ \text{of bellows}\end{pmatrix}\times(\text{Displacement})$$

The spring-stiffness and the bellows area are both constants and therefore the bellows is a proportional transducer.

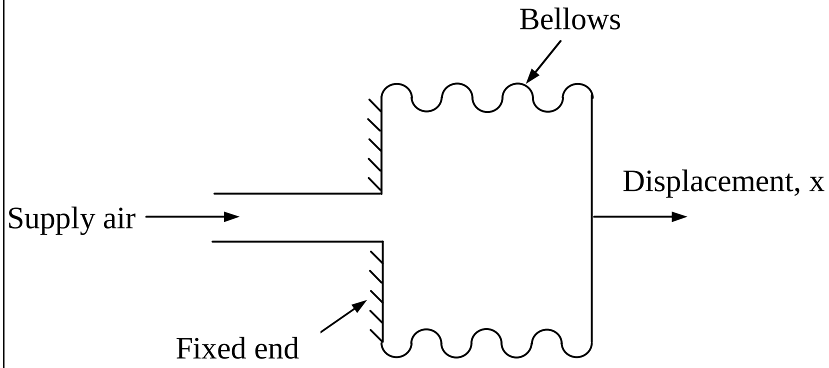


Figure 6 Bellows mechanism

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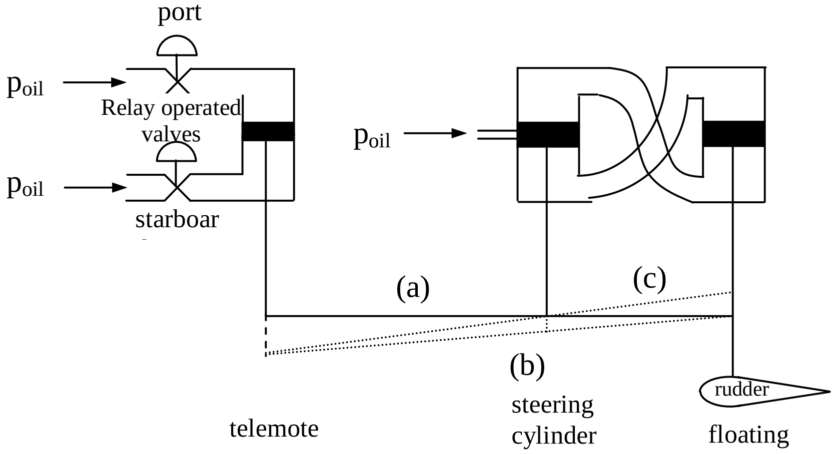
| | In some feedback arrangements a restrictor is fitted to the air supply to the bellows. The effect of this will be to introduce a time delay into the operation of the bellows. This time delay will be related to the size of the restriction and the capacitance of the bellows.<br><br>In practise it is usual for bellows to be made of brass with a low spring-stiffness and to insert a spring. The displacement may therefore be increased, and also the effects of any pressure variations. | |
| | <u>(d) A final control element may be a hydraulic actuator in steering machine:</u> Where a flowing liquid is used as the operation medium, this can be generally considered as hydraulic control. Hydraulics is, however, usually concerned with the transmission of power, rather than the transmission of signals. | 2<br><br><br><br>Sub: 8 |
| | Hydraulic systems enable the transfer of power over large distances with infinitely variable speed control of linear and rotary motions. High static forces or torques can be applied and maintained for long periods by compact equipment. The equipment itself is safe and reliable, and overload or supply failure situations can be safeguarded against. Hydraulic operation of a ship's steering gear is usual and use is often made of hydraulic equipment for both mooring and carriage handling deck machinery.<br><br>Hydraulic systems utilize pumps, valves, motors or actuators and various ancillary fittings. The system components can be interconnected in a variety of different circuits. Using their low or medium present oil. Example of a hydraulic control system (Ship Steering Machine):<br><br><br>Figure 7 Simplified diagram of a two stage hydraulic steering machine | |

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| **03** | (a) The differential equation is rewritten as<br><br>$$\ddot{y} = -\frac{b}{a}\dot{y} + \frac{c}{a}y + \frac{d}{a}u \quad (a \neq 0)$$<br>Substituting $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, where $\begin{matrix} x_1 = y \\ x_2 = \dot{y} \end{matrix}$<br><br>yields the state space representation as follows<br><br>$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}$$<br>where $A = \begin{bmatrix} 0 & 1 \\ -c/a & -b/a \end{bmatrix}$; $B = \begin{bmatrix} 0 \\ d/a \end{bmatrix}$ and $C = [1 \quad 0]$. | 2 |
| | (b) Taking Laplace transforms of two sides of the differential equation with zero initial conditions, yields<br>$as^2 y(s) + bsy(s) + cy(s) = du(s)$<br>The transfer function is<br><br>$$H(s) = \frac{y(s)}{u(s)} = \frac{d}{as^2 + bs + c}$$<br>$$H(s) = \frac{y(s)}{u(s)} = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$<br>where $2\zeta\omega_n = \frac{b}{a}$, $\omega_n^2 = \frac{c}{a}$ and $K = \frac{d}{a}$.<br>In case of a = 2, b = 4, c = d = 13 (K = 13/2 = 6.5, $\omega_n = \sqrt{6.5}$,<br>$\zeta = \frac{1}{\sqrt{6.5}}$<br>The transfer function has no zeros, and has poles at<br>$$s = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} = -1 \pm \sqrt{6.5}\sqrt{\frac{1}{6.5} - 1}$$<br>$(-1 \pm 2.3452j)$.<br><br>The closed-loop transfer function is<br>$$F(s) = \frac{H}{1+H} = \frac{\frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}}{1 + \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}} = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2 + K}$$ | 2 |

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| | $F(s) = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2 + K} = \frac{6.5}{s^2 + 2s + 13}$<br>The closed-loop system transfer function has two poles at $s = -1 \pm \sqrt{12}j$ that are in the left-half s-plane, so the closed-loop system is stable.<br><br>(c) The steady state error is determined as follows:<br>$E = R(1-H) = R\left(1 - \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}\right) = \frac{1}{s^2}\frac{s^2 + 2\zeta\omega_n s + \omega_n^2 - K}{s^2 + 2\zeta\omega_n s + \omega_n^2}$<br><br>If $\omega_n^2 = K$, the steady stead error is:<br>$SSE = \lim_{s\to 0} sE = \lim_{s\to 0} s\frac{1}{s^2}\frac{s^2 + 2\zeta\omega_n s}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{2\zeta}{\omega_n}$<br>If $\omega_n^2 \neq K$, the steady stead error does not exist because of<br>$SSE = \lim_{s\to 0} sE = \lim_{s\to 0} s\frac{1}{s^2}\frac{s^2 + 2\zeta\omega_n s + \omega_n^2 - K}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \infty$<br><br>(d) The closed-loop transfer function is:<br><br>$F(s) = \frac{CH}{1+CH} = \frac{K_P K}{1 + K_P \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}} = \frac{K_P K}{s^2 + 2\zeta\omega_n s + \omega_n^2 + K_P K}$<br><br>$F(s) = \frac{13K_P}{s^2 + 2s + 13 + 13K_P}$<br>Applying the Routh-Hurwitz criterion, yields<br>$s^2$ 1 $13 + 13K_P$<br>$s^1$ 2 0<br>$s^0$ $13 + 13K_P$ 0<br>If the closed-loop control stable, $13 + 13K_P$ must be greater than zero, i.e. $K_P > -1$. However, in practice, $K_P$ is often a positive, so in this case of $K_P > 0$ the system is stable. | 2<br><br>2<br><br>Sub: 8 |
| **04** | (a) <u>Static Performance:</u> When steady or constant input signals are applied, comparison of the steady output with the ideal case gives the static performance of the system;<br><br><u>Accuracy and precision:</u> Accuracy is normally stated in terms of | 4 |

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| | the errors introduced, where<br>$$\text{Percentage error} = \frac{\text{indicated} - \text{true value}}{\text{true value}} \times 100\%$$<br>However, it is common practice to express the error as percentage of the measuring range of the equipment:<br>$$\text{Percentage error} = \frac{\text{indicated value} - \text{true value}}{\text{maximum scale value}} \times 100\%$$<br>Precision is a term that is sometimes confused with accuracy, but a precise measurement may not be an accurate measurement. If the measuring device is subjected to the same input on a number of occasions and the indicated results lie closely together, then the instrument is said to be of high precision. The term used to specify the closeness of results is the reproducibility of the instrument. (*Examples may be taken here*).<br><br><u>Reproducibility:</u> A general term applied to the ability of a measuring system or instrument to display the same reading for a given input applied on a number of occasions.<br><br><u>Repeatability:</u> The reproducibility when a constant input is applied repeatedly at short intervals of time under fixed conditions of use.<br><br><u>Stability:</u> The reproducibility when a constant input is applied over long periods of time compared with the time of taking a reading, under fixed conditions of use.<br><br><u>Span:</u> The range of input signals corresponding to the designed working range of the output signal.<br><br><u>Linearity:</u> The maximum deviation from a linear relationship between input and output, i.e. from a constant sensitivity – expressed as a percentage of full scale. | |
| | (b) <u>Dynamic Performance:</u> when changing input signals are applied, comparison with the ideal case gives the dynamic performance of the system. | 4 |
| | From the given differential equation representing the measuring system, the transfer function is:<br>$$H(s) = \frac{Y(s)}{U(s)} = \frac{K}{\tau s + 1}$$<br>(i) When unit step signal is applied, the Laplace-transform of the output response is:<br>$$Y(s) = \frac{K}{\tau s + 1}\frac{1}{s}$$ | Sub: 8 |

The output response is illustrated as follows:

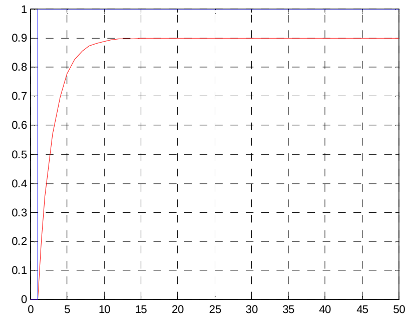


Figure 8 Step response of a first-order system

The steady state error is defined as:

$$SSE = \lim_{s\to\infty} eE = \lim_{s\to\infty} s\frac{1}{s}(1 - CLTF) = \lim_{s\to\infty} s\frac{1}{s}\left(1 - \frac{H}{1+H}\right)$$

$$SSE = \lim_{s\to\infty} sE = \lim_{s\to\infty} s\frac{1}{s}\left(\frac{1}{1+\frac{K}{\tau s+1}}\right) = \lim_{s\to\infty} s\frac{1}{s}\frac{\tau s+1}{\tau s+1+K} = \frac{1}{1+K}$$

(ii) When unit ramp test signal is applied, the Laplace-transform of the output response is:

$$Y(s) = \frac{K}{\tau s+1}\frac{1}{s^2}$$

The output response in case of K = 1 is illustrated as follows:

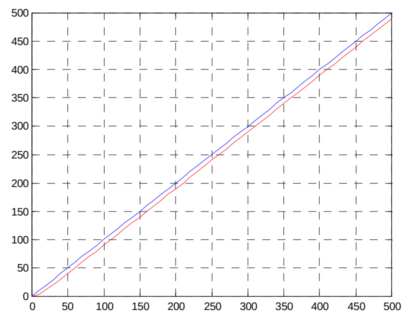


Figure 9 Ramp response of a first-order system

The steady state error is defined as

$$SSE = \lim_{s\to\infty} sE = \lim_{s\to\infty} s\frac{1}{s}(1 - CLTF) \text{ where } CLTF = \frac{H}{1+H}$$

$$\lim_{s\to\infty} s\frac{1}{s}\left(1 - \frac{H}{1+H}\right) = \lim_{s\to\infty} s\frac{1}{s^2}\left(\frac{1}{1+H}\right)$$

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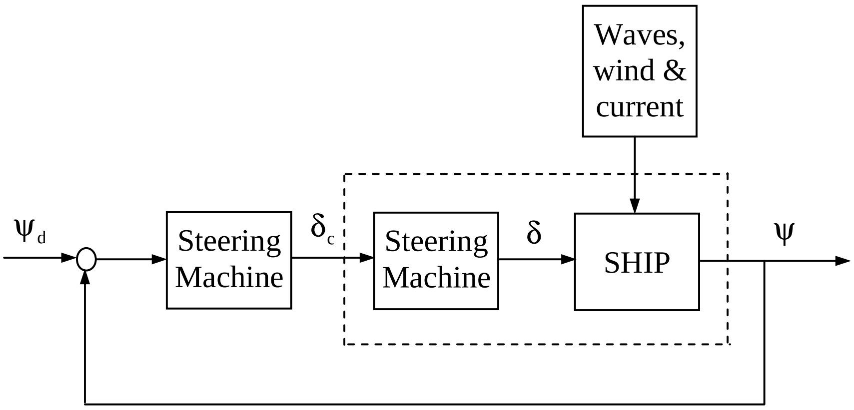
| | $\lim_{s\to\infty} s\frac{1}{s^2}\left(\frac{1}{1+\frac{K}{\tau s+1}}\right) = \lim_{s\to\infty}\frac{1}{s}\frac{\tau s+1}{\tau s+1+K} = \infty$ | |
| **05** | <u>(a) Description of Autopilot System:</u> Autopilots for course-keeping are normally based on feedback from a gyrocompass measuring the heading. Heading rate measurements can be obtained by a rate sensor, gyro, numerical differentiation of the heading measurement or a state estimator. This is common practice in most control laws utilizing proportional, derivative and integral action. The control objective for a course-keeping autopilot can be expressed as<br><br>$\psi_d$ = constant<br><br>This is illustrated in Figure 5.1. On the contrary, course-changing manoeuvres suggest that the dynamics of the desired heading should be considered in addition.<br><br><br><br>Figure 10 Autopilot for automatic heading<br><br><u>(b) Principle:</u> PID-Control Typed Autopilot<br><br>During autopilot control of a ship it is observed that a rudder off-set is required to maintain the ship on constant course. The reason for this is a yaw moment caused by the rotating propeller and the slowly-varying environmental disturbances. These are wave drift forces (2nd-order wave disturbances) and LF components of wind and sea currents. However, steady-state errors due to wind, current and wave drift can all be compensated for by adding integral | 4<br><br>4<br><br>Sub: 8 |

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| | action to the control law. Consider the PID-control law: $$\delta = K_P(\psi_d - \psi) - K_D\dot{\psi} + K_I\int_o^t(\psi_d - \psi(\tau))d\tau \quad (5.1)$$ where $K_P > 0$, $K_D > 0$ and $K_I > 0$ are the regulator design parameters. Applying this control law to Nomoto's 1st-order model: $$T\ddot{\psi} + \dot{\psi} = K(\delta - \delta_0) \quad (5.2)$$ where $\delta_0$ is the steady-state rudder off-set, yields the following closed-loop characteristic equation $$T\sigma^3 + (1 + KK_D)\sigma^2 + KK_P\sigma + KK_I = 0 \quad (5.3)$$ Hence the triple ($K_P$, $K_D$, $K_I$) must be chosen such that all the roots of this 3rd-order polynomial become negative, that is $$\mathrm{Re}\{\sigma_i\} < 0 \text{ for } (i = 1, 2, 3) \quad (5.4)$$ This can be done by applying Routh's stability criterion. Another simple intuitive way to do this is by noticing that $\delta$ can be written as: $$\delta = K_P\left(1 + T_D s + \frac{1}{T_I s}\right)(\psi_d - \psi) \quad (5.5)$$ where the derivative and integral time constants are $T_D = K_D/K_P$ and $T_I = K_P/K_I$, respectively. Hence, integral action can be obtained by first designing the PD-controller gains $K_D$ and $K_P$ according to the previous discussions. This ensures that sufficient stability is obtained. The next step is to include integral action by adjusting the integral gain $K_I$. A rule of thumb can be to choose: $$\frac{1}{T_I} \approx \frac{\omega_n}{10} \quad (5.6)$$ which suggests that $K_I$ should be chosen as: $$K_I = \frac{\omega_n}{10}K_P = \frac{\omega_n^3}{10}\frac{T}{K} \quad (5.7)$$ | |
| **06** | (a) PID stands for Proportional, Integral and Derivative. PID control is a type of control consisting of three control actions: Proportional control, Integral control and Derivative control. Now let's consider the following system: | 4 |

**Control Actions**:

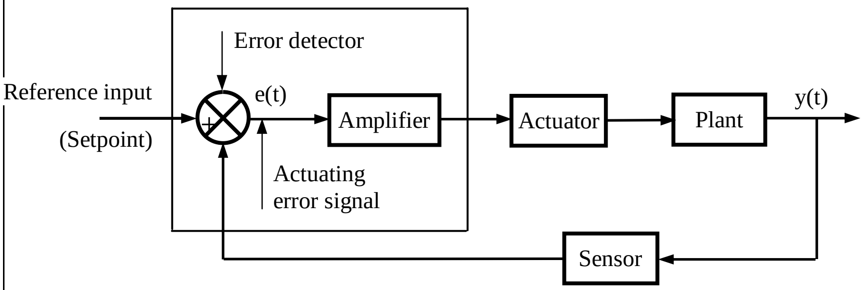


Figure 11 Basic control actions and response of control systems

**Proportional action (P)**: For a controller with proportional control action, the relationship between the output of the controller y(t) and the actuating error signal e(t) is

$$y(t) = K_P e(t) \qquad (6.1)$$

or, in Laplace transformed quantities,

$$\frac{Y(s)}{E(s)} = K_P \qquad (6.2)$$

where $K_p$ is termed the proportional gain.

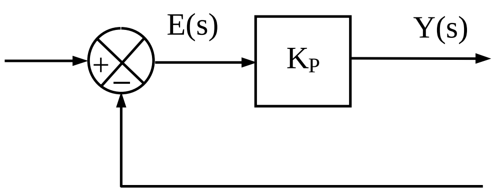


Figure 12 Block diagram of a proportional controller

What ever the actual mechanism may be and whatever the form of the operating power, the proportional controller is essentially an amplifier with an adjustable gain. A block diagram of such a controller is shown in Figure 2.26.

**Integral control action (I)**: In a controller with integral control action, the value of the controller output u(t) is changed at a rate proportional to the actuating error signal e(t). That is,

$$\frac{dy(t)}{dt} = K_I e(t) \quad (6.3)$$

thus

$$y(t) = K_I \int_0^t e(t)dt \quad (6.4)$$

where $K_I$ is an adjustable constant. The transfer function of the integral controller is

$$\frac{Y(s)}{E(s)} = \frac{K_I}{s} \quad (6.5)$$

If the value of e(t) is doubled, then the value of y(t) varies twice as fast. For zero actuating error, the value of y(t) remain stationary. The integral control action is sometimes called reset control. Figure 13 shows a block diagram of such a controller.

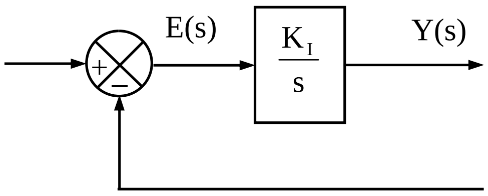


Figure 13 Block diagram of an integral controller

**<u>Derivative control action (D)</u>**. In a controller with derivative control action, the value of the controller output u(t) is changed at a rate proportional to the rate of the change of the actuating error signal e(t). That is,

$$y(t) = K_D \frac{de(t)}{dt} \quad (6.6)$$

or transfer function of the controller is

$$\frac{Y(s)}{E(s)} = K_D s \quad (6.7)$$

Note that the derivative control action can never be used alone because this control action is effective only during transient periods. See the proportional derivative control action.

**Proportional Integral control action (PI)**. The control action of proportional integral controller known as PI controller is defined by:

$$y(t) = K_P e(t) + \frac{K_P}{T_I}\int_0^t e(t)dt \qquad (6.8)$$

or the transfer function of the controller is

$$\frac{Y(s)}{E(s)} = K_P\left(1 + \frac{1}{T_I s}\right) \qquad (6.9)$$

where $K_P$ is the proportional gain, and $T_I$ is called the integral time. Both $K_P$ and $T_I$ are adjustable. The integral time adjusts the integral control action, while a change in the value of $K_P$ affects both the proportional and integral parts of the control action. The inverse of the integral time $T_I$ is called the reset rate. The reset rate is the number of times per minute that the proportional part of the control action is duplicated. Reset rate is measured in terms of repeats per minute. Figure 14(a) shows a block diagram of a proportional-integral controller. If the actuating error signal e(t) is a unit step function as shown in Figure 14(b), then the controller output y(t) becomes as shown in Figure 14(c).

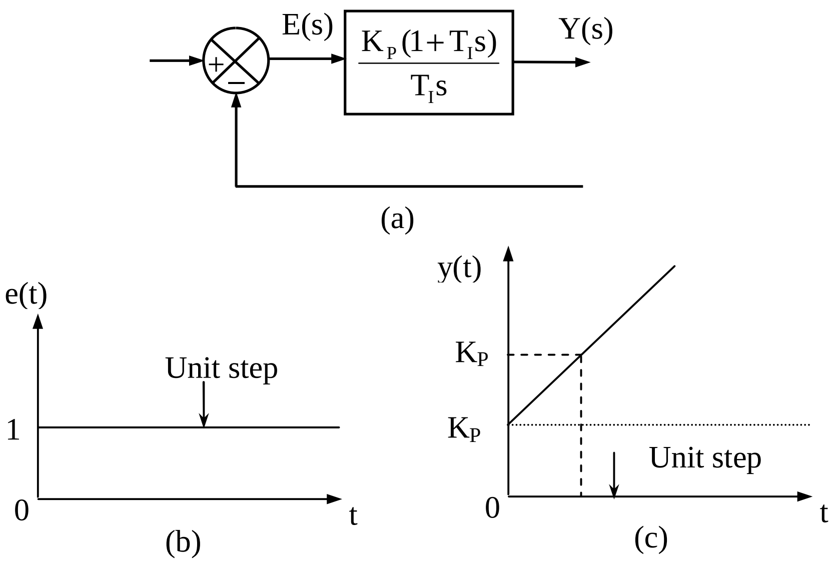


Figure 14 (a) block diagram of a proportional integral controller; (b) & (c) diagrams depicting a unit-step input and the controller output

***Proportional-derivative control action***. The control action of a proportional-derivative controller is defined by

$$y(t) = K_P e(t) + K_P T_D \frac{de(t)}{dt} \qquad (6.10)$$

and the transfer function is

$$\frac{Y(s)}{E(s)} = K_P e(t) + K_P T_D \frac{de(t)}{dt} \qquad (6.11)$$

where $K_P$ is the proportional gain and $T_D$ is a constant called the derivative time. Both $K_P$ and $T_D$ are adjustable. The derivative control action, sometimes called rate control, is where the magnitude of the controller output is proportional to the rate of change of the actuating error signal. The derivative time $T_D$ is the time interval by which the rate action advances the effect of the proportional control action. Figure 15(a) show a block diagram of a proportional-derivative controller. If the actuating error signal e(t) is unit-ramp function as shown in Figure 15(b), then the controller output y(t) becomes as shown in Figure 15(c). As may be seen from Figure 15(c), the derivative control action can never anticipate any action that has not yet taken place.

While derivative control action has the advantage of being anticipatory, it has the disadvantages that it amplifies signals and may cause a saturation effect in the actuator.

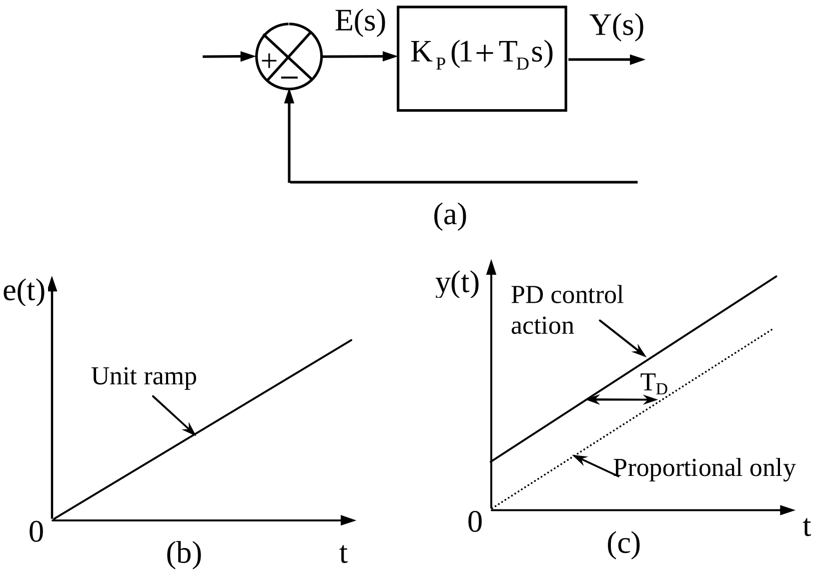


Figure 15 (a) block diagram of a proportional derivative controller; (b) & (c) diagrams depicting a unit-ramp input and the controller output

**PID control is a widely used control system.**

**What is PID control?**

Proportional-Integral and Derivative Control Action (three term control). The combination of proportional control action, integral control action, and derivative control action is termed proportional, integral and derivative control action, known as PID control. This combined action has the advantages of each of the three individual control actions. The equation of a controller with this combined action is given by

$$y(t) = K_P e(t) + \frac{K_P}{T_I}\int_0^t e(t)dt + K_P T_D \frac{de(t)}{dt} \qquad (6.12)$$

or transfer function is

$$\frac{Y(s)}{E(s)} = K_P\left(1 + \frac{1}{T_I s} + T_D s\right) \qquad (6.13)$$

where $K_P$ is the proportional gain, $T_I$ is the integral time, and $T_D$ is the derivative time. The block diagram of a proportional, integral and derivative controller is shown in Figure 16(a). If e(t) is a unit-ramp function as shown in Figure 16(b), then the controller output y(t) becomes as shown in Figure 2.30(c).

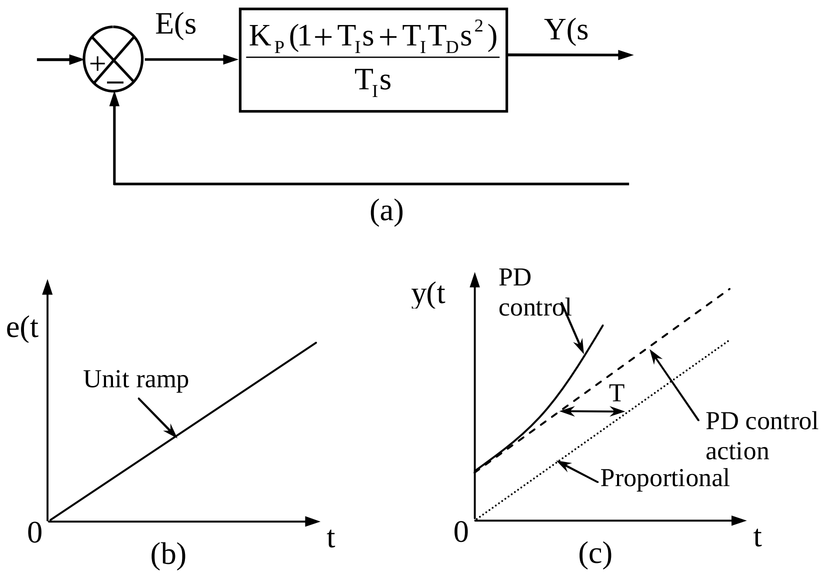


Figure 16 (a) block diagram of a PID controller; (b) and (c) diagrams depicting a unit-ramp input and the controller output

(b)

(i) The closed-loop transfer function is

$$F(s) = \frac{CH}{1+CH} = \frac{\left(K_P + \frac{1}{T_I s}\right)\left(\frac{K}{Ts+1}\right)}{1 + \left(K_P + \frac{1}{T_I s}\right)\left(\frac{K}{Ts+1}\right)} = \frac{(K_P T_I s + 1)K}{T_I s(Ts+1) + (K_P T_I s + 1)K}$$

4

Sub: 8

Substituting K = 0.11, T = 7.5, $K_P$ = 4, and $T_I$ = 100 seconds, yields

$$F(s) = \frac{(4\times100s+1)0.11}{100s(7.5s+1)+(4\times100s+1)0.11} = \frac{(400s+1)0.11}{750s^2+500s+0.01}$$

$$C = K_P + \frac{1}{T_I s} = \frac{K_P T_I s+1}{T_T s}\text{; or } C = K_P + \frac{K_I}{s},\ K_I = \frac{1}{100} = 0.01$$

$$F(s) = \frac{CH}{1+CH} = \frac{\frac{K_P T_I s+1}{T_I s}\frac{K}{Ts+1}}{1+\frac{K_P T_I s+1}{T_I s}\frac{K}{Ts+1}} = \frac{(K_P T_I s+1)K}{T_I s(Ts+1)+(K_P T_I s+1)K} = \frac{(400s+1)0.11}{750s^2+144s+0.11}$$

Zeros: s = - 1/400 = -0.00025

Poles: s = -0.1912 & s = -0.0008

The open-loop transfer function is

$$L(s) = CH = \left(K_P + \frac{1}{T_I s}\right)\left(\frac{K}{Ts+1}\right) = \frac{(K_P T_I s+1)K}{T_I s(Ts+1)}$$

(ii) The closed-loop transfer function is

$$F(s) = \frac{CH}{1+CH} = \frac{(K_P s+K_I)K}{s(Ts+1)+(K_P s+K_I)K}$$

$$F(s) = \frac{CH}{1+CH} = \frac{(K_P T_I s+1)K}{T_I s(Ts+1)+(K_P T_I s+1)K}$$

Substituting values of K, $K_P$, T and $T_I$, yields:

$$F(s) = \frac{(400s+1)0.11}{750s^2+144s+0.11}$$

The characteristic equation is:

CE: $750s^2+144s+0.11=0$

Applying the Routh-Hurwitz criterion (the CE satisfies the necessary and sufficient conditions), yields:

| | | |
|---|---|---|
| $s^2$ | 750 | 0.11 |
| $s^1$ | 144 | 0 |
| $s^0$ | 0.02112 | 0 |

No sign change in the first column, the closed-loop is stable.
(*Nyquist stability criterion can be applied by plotting the Nyquist diagram*)

| **TOTAL MARKS** | **48** |
|---|---|

*Notes: The above-mentioned answers are the complete solutions to the final examination paper. Students can get the maximum marks if they give the outlined correct answers to all questions.*