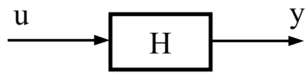
AUSTRALIAN MARITIME COLLEGE

SEMESTER 2 - SUPPLEMENTARY EXAM ON 18th FEB 2004

SOLUTIONS TO SUPPLEMENTARY EXAMINATION PAPER

SUBJECT INSTR & PROCESS CONTROL TIME ALLOWED 3 HOURS

NO OF QUESTIONS TO BE ATTEMPED SIX (6)

QUESTION NO	SOLUTION	MARK ANALYSIS
01	**QUESTION 1** *Q: (a) State the open-loop control system and its features. Give an example of the open-loop control system.* A: (a) Those systems in which the output has no effect on the control action are called open-loop control systems. In other words, in an open-loop control system the output is neither measured nor fed back for comparison with the input. The open-loop control system can be expressed by the following block diagram (u is input and y is output).  Figure 1 Block diagram of an open-loop control system Features: Output is not compared with the reference input. It has the following properties: Advantages: Relatively simple, resulting in cost, reliability and maintainability advantages and inherently stable; Disadvantages: Relatively slow in response to demanded changes Inaccurate, due to lack of corrective action for error (that is, departure of actual value from desired value). Example: One practical example is a washing machine. Soaking, washing, and rinsing in the washer operate on a time basis. The machine does not measure the output signal, that is, the cleanliness of the clothes. *Q: (b) State the closed-loop control system and its features. Give an example of the closed-loop control system.* A: (b) A system that maintains a prescribed relationship between the output and the reference input by comparing them and using the difference as a means of control is called a feedback control	 [2 Marks]

system, or closed-loop control system. A closed-loop control system can be expressed in the following block diagram (R: reference signal, Y: output signal and E: error (difference) signal.

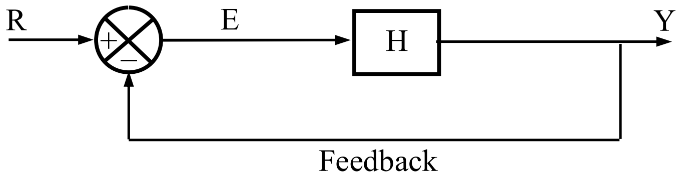


Figure 2 Block diagram of a closed-loop control system

Features: In the closed-loop control system, output is compared with the reference input. It has the following properties:
Advantages: Relatively fast in response to demanded changes, and relatively accurate in matching actual to desired value;
Disadvantages: Relatively complex, and potentially unstable, under fault conditions
Example: An example would be a room-temperature control system. By measuring the actual room temperature and comparing it with the reference temperature (desired temperature), the thermostat turns the heating or cooling equipment on or off in such a way as to ensure that the room temperature remains at a comfortable level regardless of outside conditions.

[2 Marks]

Q: (c) Explain the term "Steady State Error" (SSE) of a system. State the final value theorem and apply to find the SSE of a second-order system. Use diagrams to illustrate your answer.

A: (c) Steady State Error of a system is the difference between the output and the reference in the steady state. See the following graphs.

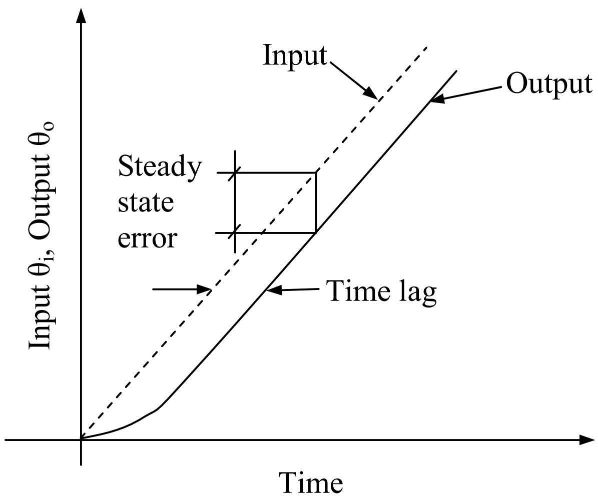


Figure 3 Concept of steady state error (SSE)

	The final value theorem states that the final value of a time varying function can be found (if it exists) by taking the Laplace-transform of the time function, multiplying that by "s" and then let s tend to zero: $$\text{If } \lim_{t\to\infty} e(t) \text{ exists, then } \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$ This theorem can be applied to automatic control systems to find the value of the steady state error which will be the final value of 'Set Point – Process Variable'. Therefore, the SSE can be found as follows: $$SSE = \lim_{s\to 0} sE(s)$$ *Q: (d) Explain an ideal measuring system. Explain the necessity for calibration of a measuring system.* A: (d) An ideal measuring system is one where the output signal has a linear relationship with the measurand, where no errors are introduced by effects such as static fiction, and where the output is a faithful reproduction of the input no matter how the input varies. This is, of course, a theoretical case and serves only a comparison for actual results obtained from a measurement. Failure of a measuring system or instrument to match up to the perfect case is usually specified in terms of errors, where error is defined as the difference between the indicated value and the "true value." (*J.A. Haslam et al, Engineering Instrumentation and Control*) Necessity for calibration of a measuring system: The adoption of a fundamental set of standards and units provides a common upon which measurement can be made. If these fundamental or reference standards are natural ones – definable in atomic process terms – they may be accurately reproduced anywhere in the world other than being kept in one particular laboratory under special conditions. The process of comparing an instrument with a known standard is referred to as "calibration". Calibration is normally performed either by varying one input quantity with all other parameters kept constant and observing the resulting output variations or, possibly, by marking or graduating an output scale as the primary quantity is varied through its full range. In practice a "primary standard" instrument is used to calibrate a secondary or working-standard instrument which in turn is used to calibrate the device in use. The accuracy and calibration of each device is therefore traceable back to the fundamental standard via the secondary and primary standards.	[2 Marks] [2 Marks]
02	**QUESTION 2** **Q: (a) Define the transfer function (continuous-time and**	

discrete-time) of a dynamic system, its poles and zeros. Use examples to illustrate your answer.

A: (a) Transfer function of a system is ratio of the Laplace transform of the output signal to the Laplace transform of the corresponding input signal. Given the following system with input U(s) and output Y(s), the transfer function is defined as follows:

$$H(s) = \frac{Y(s)}{U(s)} \text{ or } Y(s) = H(s) \times U(s)$$

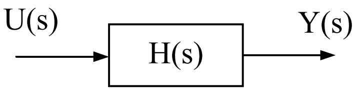


Figure 4 Transfer function definition

A transfer function is often a fraction of two polynomial (numerator and denominator polynomials and represented by the following expression:

$$H(s) = \frac{N}{D} = \frac{b_n s^n + b_{n-1} s^{n-1} + ... + b_1 s + b_0}{a_m s^m + a_{m-1} s^{m-1} + ... + as + a_0} \quad (m \geq n)$$

For the discrete-time system, the transfer function is defined as:

$$H(z^{-1}) = \frac{N}{D} = \frac{Y(z^{-1})}{U(z^{-1})} = \frac{b_0 + b_1 z^{-1} + ... + b_{n-1} z^{n-1} + b_n z^{-n}}{a_0 + a_1 z^{-1} + ... + a_{m-1} z^{m-1} + a_m z^{-m}} \quad (m \geq n)$$

where z^{-i} is the backward shift operator $z^{-i} y(k) = y(k-i)$. The discrete-time transfer function can be expressed by a fraction of two polynomial of z (the forward shift operator, $z^i y(k) = y(k+i)$) as follows:

$$H(z) = \frac{N}{D} = \frac{Y(z)}{U(z)} = \frac{b_n z^n + b_{n-1} z^{n-1} + ... + b_1 z + b_0}{a_m z^m + a_{m-1} z^{m-1} + ... + az + a_0} \quad (m \geq n)$$

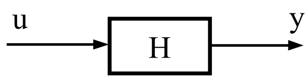
Poles of a transfer function are the roots of the equation D = 0.
Zeros of a transfer function are the roots of the equation N = 0.
Examples:

Continuous transfer function: $H(s) = \frac{(s+1)K}{s^2 + 5s + 12}$ has zeros at s = -1, and poles at $s = \frac{-5 \pm \sqrt{23} j}{2}$

Discrete-time transfer function: $H(z) = \frac{(z+1)K}{z^2 + 5z + 7}$ has zeros at z = -1 and poles at $z = \frac{-5 \pm \sqrt{3} j}{2}$.

[4 Marks]

	Q: (b) What is the static performance of a measuring system? Define the following terms and show examples of them: sensitivity, accuracy and precision, possible and probable errors, reproducibility, repeatability, stability, span, linearity. Use examples (where applicable) to illustrate your answer. A: (b) Concept of the static performance of a measuring system: when steady or constant input signal applied, comparison of the steady output with the ideal case gives the static performance of the system. To examine the static performance of a measuring system, the following terms are often used: sensitivity, accuracy and precision, possible and probable errors, reproducibility, repeatability, stability, span and linearity. <u>Sensitivity:</u> Sensitivity of a measuring system is the ratio of the change in output to the corresponding change in input under static or steady-state conditions. Sensitivity is represented by the following formula: $$K = \frac{\Delta y}{\Delta u}$$ where Δy and Δu are the change in output and the change in input, respectively. Unit of the sensibility depends on the instrument or measuring system being considered. <u>Accuracy and precision:</u> Accuracy is normally stated in terms of the errors introduced, where $$\text{Percentage error} = \frac{\text{indicated value} - \text{true value}}{\text{true value}} \times 100\%$$ It is common practice to express the error as a percentage of the measuring range of the instrument as follows: $$\text{Percentage error (full scale)} = \frac{\text{indicated value} - \text{true value}}{\text{maximum scale value}} \times 100\%$$ Precision is a term that is used to specify the closeness of results is the reproducibility of the instrument. If the measuring device is subjected to the same input on a number of occasions and the indicated results lie closely together, then the instrument is said to be of high precision. The concepts of accuracy and precision are illustrated by the following figures. (*three figures (high accuracy & high precision, low accuracy & high precision, low accuracy and low precision) here*) <u>Possible and probable errors:</u> The maximum possible error is the sum of individual errors of a measuring system (*example*). The probable error is defined as the square root of the sum of squares of the individual errors (*example*). <u>Reproducibility:</u> A general term applied to the ability of a measuring system or instrument to display the same reading for a	

	given input applied on a number of occasions. Repeatability: The reproducibility when a constant input is applied repeatedly in short intervals of time under fixed conditions of use. Stability: The reproducibility when a constant input is applied over long periods of time compared with the time of taking a reading, under fixed conditions of use. Range: The total range of values which an instrument or measuring system is capable of measuring. Span: The range of input signals corresponding to the designed working range of the output signal. Constancy: The reproducibility when a constant input is presented continuously and the conditions of test are allowed to vary within specified limits, due to some external effect such as a temperature variation. Linearity: the maximum deviation from a linear relationship between input and output – from a constant sensitivity – expressed as a percentage of full scale.	[4 Marks]
03	**QUESTION 3** **Q: The following differential equation represents a dynamic system** $a\ddot{y} + b\dot{y} + cy = du$ *where a, b, c and d are constants, y and u are output and input as shown in Figure 1.*  Figure 1 *(a) Write the state space representation for the system.* A: (a) The differential equation is rewritten as $\ddot{y} = -\frac{b}{a}\dot{y} + \frac{c}{a}y + \frac{d}{a}u \quad (a \neq 0)$ Substituting $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, where $\begin{matrix} x_1 = y \\ x_2 = \dot{y} \end{matrix}$ yields the state space representation as follows	

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

where $A = \begin{bmatrix} 0 & 1 \\ -c/a & -b/a \end{bmatrix}$; $B = \begin{bmatrix} 0 \\ d/a \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$.

[2 Marks]

Q: (b) Find the transfer function (H, with zero initial conditions) of the system. Find poles and zeros of the transfer function in case of a = 1, b = 5, and c = d = 7. Then determine if the closed loop system shown in Figure 2 is stable.

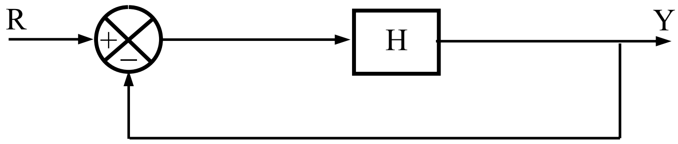


Figure 2

A: (b) Taking Laplace transforms of two sides of the differential equation with zero initial conditions, yields

$$as^2 y(s) + bsy(s) + cy(s) = du(s)$$

The transfer function is

$$H(s) = \frac{y(s)}{u(s)} = \frac{d}{as^2 + bs + c}$$

$$H(s) = \frac{y(s)}{u(s)} = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where $2\zeta\omega_n = \frac{b}{a}$, $\omega_n^2 = \frac{c}{a}$ and $K = \frac{d}{a}$.

In case of a = 1, b = 5, c=d=7 (K = 7/1 = 7, $\omega_n = \sqrt{7}$, $\zeta = \frac{5}{2\sqrt{7}}$)

The transfer function has no zeros, and has poles at

$$s = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} = -1 \pm \sqrt{7}\sqrt{\frac{25}{4\times 7} - 1} = -1 \pm \frac{\sqrt{-3}}{2}$$

$(= -1 \pm 0.886j)$.

The closed-loop transfer function is

$$F(s) = \frac{H}{1+H} = \frac{\dfrac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}}{1 + \dfrac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}} = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2 + K}$$

$$F(s) = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2 + K} = \frac{7}{s^2 + 5s + 7}$$

The closed-loop system transfer function has two poles at $s = -2.5 \pm \frac{\sqrt{3}}{2} j$ that are in the left-half s-plane, so the closed-loop system is stable.

[2 Marks]

Q: (c) Find the steady state error if a unit ramp input signal is applied.

A: (c) The steady state error is determined as follows:

$$E = R(1 - CLTF) = R\left(1 - \frac{H}{1+H}\right) = R\left(\frac{1}{1+H}\right)$$

$$E = R\left(\frac{1}{1 + \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}}\right) = \frac{1}{s^2}\frac{s^2 + 2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2 + K}$$

$$SSE = \lim_{s\to 0} sE = \lim_{s\to 0}\frac{1}{s}\frac{s^2 + 2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2 + K} = \infty$$

[2 Marks]

Q: (d) In case of a = 1, b = 5, and c = d = 7 and a proportional control (C = K_P) is designed for the above system as shown in Figure 3, determine the value of K_P for which the closed-loop system is stable.

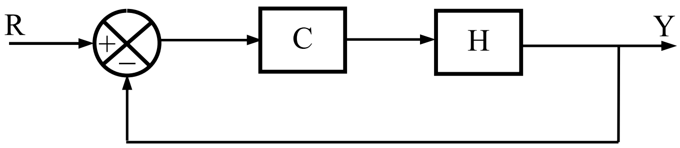


Figure 3

A: (d) The closed-loop transfer function is:

$$F(s) = \frac{CH}{1+CH} = \frac{K_P K}{1 + K_P \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}} = \frac{K_P K}{s^2 + 2\zeta\omega_n s + \omega_n^2 + K_P K}$$

$$F(s) = \frac{7K_P}{s^2 + 5s + 7 + 7K_P}$$

Applying the Routh-Hurwitz criterion, yields

s^2	1	$7+7K_P$
s^1	5	0
s^0	$7+7K_P$	0

If the closed-loop control stable, $7+7K_P$ must be greater than zero, i.e. $K_P > -1$. However, in practice, K_P is often a positive, so in this case of $K_P > 0$ the system is stable.

[2 Marks]

04

QUESTION 4

Q: (a) Explain the block diagram. State and prove the block diagram algebra.

A: (a) The relationship between an output and input of a system can be represented by a block (a rectangle) which contains a transfer function.

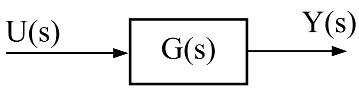


Figure 5 A block

A number of blocks interconnected by lines become a block diagram. This is effectively a shorthand representation of a control system or a part of the system.

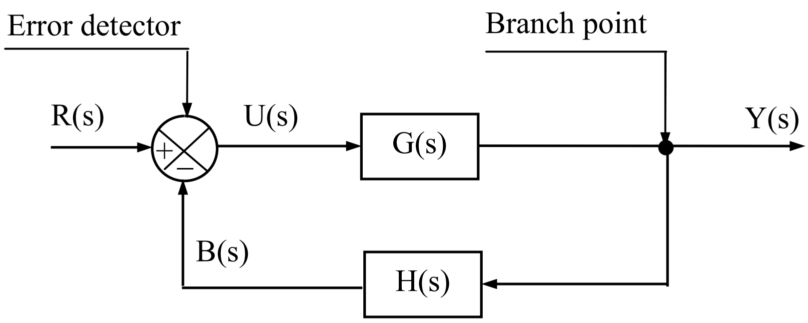


Figure 6 Block diagram of a closed-loop control system

Feed-forward transfer function, $G(s) = \dfrac{Y(s)}{U(s)}$

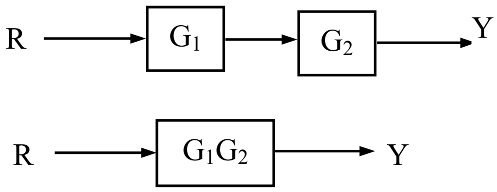
The block H(s) is in the feedback path. The ratio of the feedback signal B(s) to the actuating error signal U(s) is called the open-loop transfer function, i.e.

Open-loop transfer function, $\dfrac{B(s)}{U(s)} = G(s)H(s)$

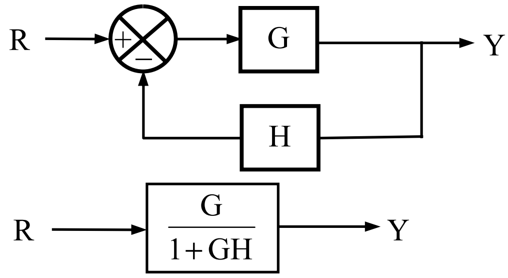
The close-loop transfer function, $\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$

Block Diagram Algebra

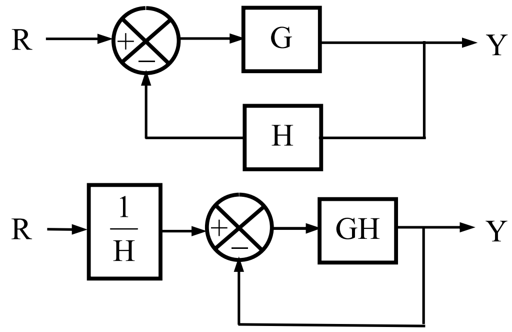
Algebra 1:



Algebra 2:



Algebra 3:



Algebra 4:

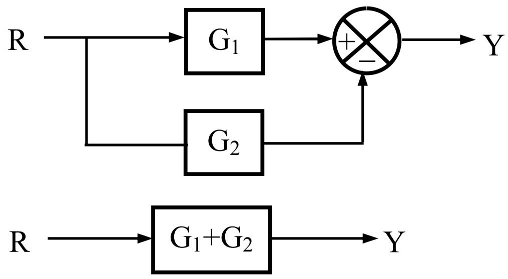


Figure 7 Block diagram algebraic

[4 Marks]

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Q: (b) Describe the general structure of a control system using the block diagram. Explain the functions of each block
A: (b) An automatic control system, including it recording (indicating) elements, can be represented by the following general block diagram.

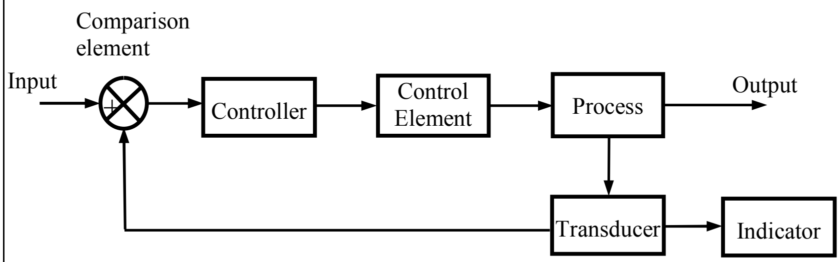

Figure 8 General block diagram of a control system

Comparison Element Block: compares the output or control variable with the desired input (reference signal) and generate an error or deviation signal to the controller. The comparison element performs the mathematical operation of subtraction.

Controller Block: calculates control signals from the error signal, and generates the control signals to the Control Element. The controller block can be a PID controller where the control signal.

Control Element Block: Control element block is the element in which the amplified and conditioned control signal is used to regulate some energy source to the process. The control element block is often referred as an actuator. Control element block may be a valve or a motor.

Process Block: is the dynamic system where the process is implemented.

Transducer Block: is a sensing device that receives the physical quantity being measured from the process, converts it into some other physical variable and generates this signal to an indicating device or feeds back to the comparison element.

Indicator Block: indicates or records the measured quantity.
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QUESTION 5

Q: (a) Describe a measuring system used in marine and offshore industries and its measuring method.
A: (a) An example of a measuring method is taken as follows:
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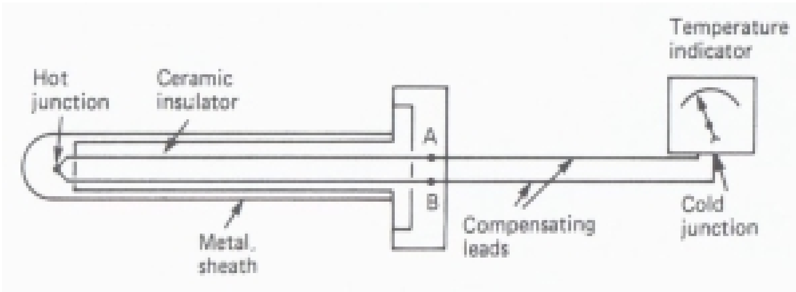
	temperature measuring system using a thermocouple. The description and principle of a thermocouple are given below. The thermocouple uses two different metals or alloys jointed together to make a closed circuit. When the two junctions are at different temperatures an e.m.f. (electromagnetic force) is generated and a current flows. The magnitude of the e.m.f. and the current flowing depend upon the temperature difference between the junctions. The arrangement used is shown in Figure 9, where extra wires or compensating leads are introduced to complete the circuit and include the indicator. As long as the two ends A and B are at the same temperature the thermoelectric effect is not influenced. The choice of metals will determine the measuring range, e.g. Copper-Constantan -200 to +350^{o}C, Platinum/Platinum and Rhodium 0 to +1500^{o}C.  Figure 9 Thermocouple *Q: (b) Describe a control system and its basic control principle. Hints: Formulas and block diagrams can be used to illustrate your answer.* A: (b) <u>Description of Autopilot System:</u> Autopilots for course-keeping are normally based on feedback from a gyrocompass measuring the heading. Heading rate measurements can be obtained by a rate sensor, gyro, numerical differentiation of the heading measurement or a state estimator. This is common practice in most control laws utilizing proportional, derivative and integral action. The control objective for a course-keeping autopilot can be expressed as ψ_d = constant This is illustrated in Figure 10. On the contrary, course-changing manoeuvres suggest that the dynamics of the desired heading should be considered in addition.	[4 Marks]

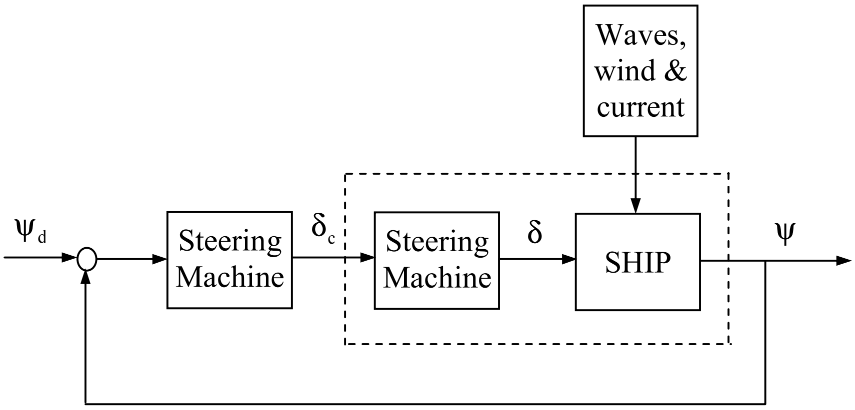


Figure 10 Autopilot for automatic heading

<u>(b) Principle:</u> PID-Control Typed Autopilot

During autopilot control of a ship it is observed that a rudder off-set is required to maintain the ship on constant course. The reason for this is a yaw moment caused by the rotating propeller and the slowly-varying environmental disturbances. These are wave drift forces (2nd-order wave disturbances) and LF components of wind and sea currents. However, steady-state errors due to wind, current and wave drift can all be compensated for by adding integral action to the control law. Consider the PID-control law:

$$\delta = K_P(\psi_d - \psi) - K_D\dot{\psi} + K_I\int_0^t(\psi_d - \psi(\tau))d\tau \qquad (5.1)$$

where $K_P > 0$, $K_D > 0$ and $K_I > 0$ are the regulator design parameters. Applying this control law to Nomoto's 1st-order model:

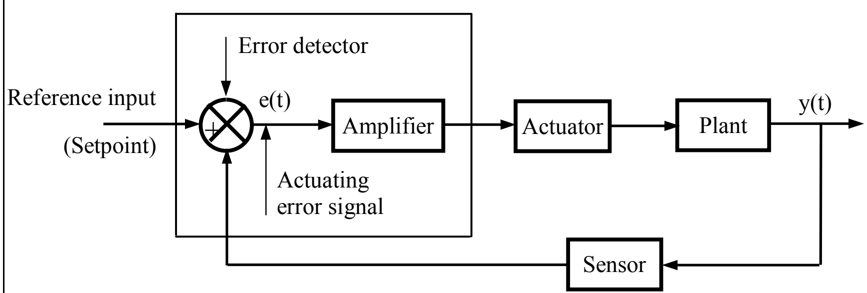
$$T\ddot{\psi} + \dot{\psi} = K(\delta - \delta_0) \qquad (5.2)$$

where δ_0 is the steady-state rudder off-set, yields the following closed-loop characteristic equation

$$T\sigma^3 + (1 + KK_D)\sigma^2 + KK_P\sigma + KK_I = 0 \qquad (5.3)$$

Hence the triple (K_P, K_D, K_I) must be chosen such that all the roots of this 3rd-order polynomial become negative, that is

$$\mathrm{Re}\{\sigma_i\} < 0 \text{ for } (i = 1, 2, 3) \qquad (5.4)$$

	This can be done by applying Routh's stability criterion. Another simple intuitive way to do this is by noticing that δ can be written as: $\delta = K_P\left(1 + T_D s + \frac{1}{T_I s}\right)(\psi_d - \psi)$ (5.5) where the derivative and integral time constants are $T_D = K_D/K_P$ and $T_I = K_P/K_I$, respectively. Hence, integral action can be obtained by first designing the PD-controller gains K_D and K_P according to the previous discussions. This ensures that sufficient stability is obtained. The next step is to include integral action by adjusting the integral gain K_I. A rule of thumb can be to choose: $\frac{1}{T_I} \approx \frac{\omega_n}{10}$ (5.6) which suggests that K_I should be chosen as: $K_I = \frac{\omega_n}{10} K_P = \frac{\omega_n^3}{10}\frac{T}{K}$ (5.7)	[4 Marks]
06	**QUESTION 6** *Q: (a) What does PID stand for? Explain the PID control action using appropriate formulae and diagrams.* A: (a) PID stands for Proportional, Integral and Derivative. PID control is a type of control consisting of three control actions: Proportional control, Integral control and Derivative control. Now let's consider the following system: **Control Actions**:  Figure 11 Basic control actions and response of control systems **What is PID control?** Proportional-Integral and Derivative Control Action (three term control). The combination of proportional control action, integral control action, and derivative control action is termed proportional, integral and derivative control action, known as PID	

control. This combined action has the advantages of each of the three individual control actions. The equation of a controller with this combined action is given by

$$y(t) = K_P e(t) + \frac{K_P}{T_I}\int_0^t e(t)dt + K_P T_D \frac{de(t)}{dt} \qquad (6.1)$$

or transfer function is

$$\frac{Y(s)}{E(s)} = K_P\left(1 + \frac{1}{T_I s} + T_D s\right) \qquad (6.2)$$

where K_P is the proportional gain, T_I is the integral time, and T_D is the derivative time. The block diagram of a proportional, integral and derivative controller is shown in Figure 12(a). If e(t) is a unit-ramp function as shown in Figure 12(b), then the controller output y(t) becomes as shown in Figure 12(c).

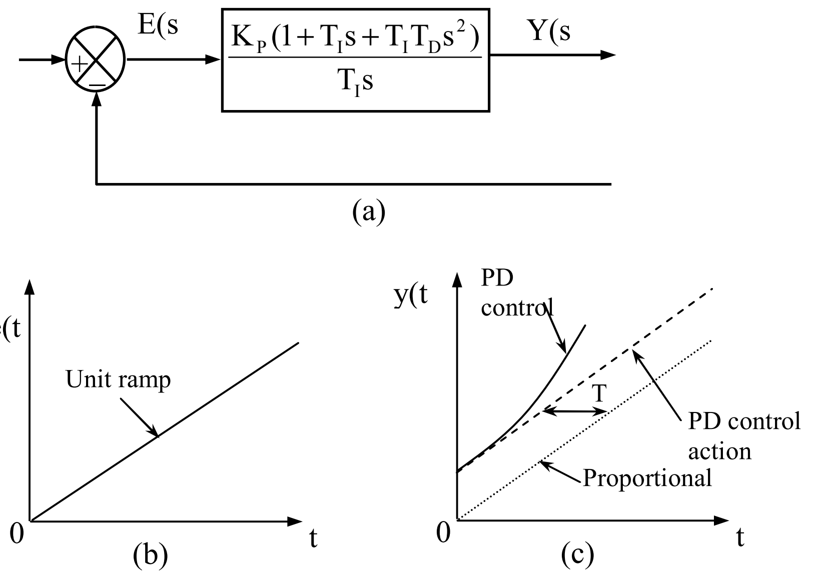


Figure 12 (a) block diagram of a PID controller; (b) and (c) diagrams depicting a unit-ramp input and the controller output

[4 Marks]

Q: (b) Given the following block diagram in Figure 4,

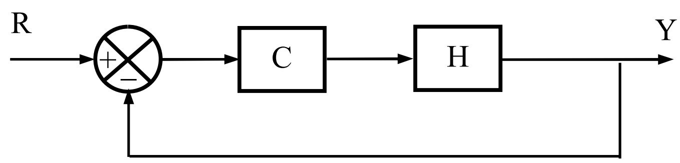


Figure 4

where $H = \dfrac{K}{Ts+1}$ *(K and T are constant, K = 0.11, T = 7.5 seconds), and* $C = K_P + T_D s$ *(K_P and T_D are proportional control gain and derivative time constant, K_P = 2, T_D = 4 seconds):*

(i) Find the closed loop transfer function and open loop transfer function. Find poles and zeros (if any) of the closed-loop transfer function.

(ii) Determine if the closed-loop system is stable.

A: (b)

(i) The closed-loop transfer function is

$$F(s) = \frac{CH}{1+CH} = \frac{(K_P + T_D s)\left(\dfrac{K}{Ts+1}\right)}{1+(K_P + T_D s)\left(\dfrac{K}{Ts+1}\right)} = \frac{(K_P + T_D s)K}{(T+T_D K)s + K_P K + 1}$$

Substituting K = 0.11, T = 7.5, K_P = 2, and T_D = 4 seconds, yields

$$F(s) = \frac{(2+4s)0.11}{(7.5+4\times 0.11)s + (2\times 0.11+1)} = \frac{(2s+1)0.11\times 2}{7.94s+1.22}$$

Zero: s = - 1/2 = -0.5

Pole: s = -1.22/7.94 = -0.154

The open-loop transfer function is

$$L(s) = CH = (K_P + T_D s)\left(\frac{K}{Ts+1}\right) = \frac{(K_P s + T_D)K}{Ts+1}$$

(ii) The closed-loop transfer function is

$$F(s) = \frac{CH}{1+CH} = \frac{(K_P + T_D s)K}{(T+T_D K)s + K_P K + 1}$$

Substituting values of K, K_P, T and T_D, yields:

$$F(s) = \frac{(2s+1)0.11\times 2}{7.94s+1.22}$$

The closed-loop characteristic equation is:

C.L.C.E: $7.94s + 1.22 = 0$

The root (s = -0.154) is real and negative, so the system is asymptotically stable.

(*The closed-loop characteristic equation is the 1st-order, so the stability can be determined straightforward.*)

[4 Marks]

Total	**48**

Notes: The above-mentioned answers are the complete solutions to the final examination paper. Students can get the maximum marks if they give the outlined correct answers to all questions.