DEPARTMENT OF MARITIME ENGINEERING

MODEL ANSWER SHEET

COURSE: BE (OE & MOS)

SUBJECT CODE: E07 267

SUBJECT DESCRIPTION: INSTRUMENTATION & PROCESS CONTROL

YEAR/SEMESTER: 2005/2

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ANSWERS	**MARKS**
QUESTION 1	**8**
(a) The main points are: Definition of transfer function (1) Formulae (1) Definition of poles of transfer function (1) Definition of zeros of transfer function (1) (Examples to illustrate) Sample answer: Transfer function representing a dynamic system is defined as the ratio of the Laplace transform of the output to the corresponding Laplace transform of the input with assumption that all initial conditions are zero. Transfer function is as follows: $G(s) = \frac{Y(s)}{U(s)}$ where Y(s) is Laplace transform of the output and U(s) is the Laplace transform of the input. The output of the dynamic system can be expressed in the following form: $Y(s) = G(s)U(s)$ Example: A dynamic system represented by the following differential equation $\ddot{y} + 2\dot{y} + 5y = u$ has the transfer function $G(s) = \frac{Y(s)}{U(s)} = \frac{1}{s^2 + 2s + 5}$ The above transfer function can be written in the form of ratio of two polynomials below: $G(s) = \frac{N(s)}{D(s)} = \frac{s^m + b_1 s^{m-1} + ... + b_{m-1}s + b_m}{s^n + a_1 s^{n-1} + ... + a_{n-1}s + a_n}$ where m is the order of the numerator polynomial N(s), and n the order of denominator polynomial D(s), n is also the order of the transfer function. Thus, poles of the transfer function are the roots of the equation D(s) = 0 or $s^n + a_1 s^{n-1} + ... + a_{n-1}s + a_n = 0$ At poles the transfer function is infinity. Zeros of the transfer function are the roots of the equation N(s) = 0 or $s^m + b_1 s^{m-1} + ... + b_{m-1}s + b_m = 0$ At zeros, the transfer function is zero. Example: The above transfer function $G(s) = \frac{Y(s)}{U(s)} = \frac{1}{s^2 + 2s + 5}$ has no zeros and two poles at $p_{1,2} = \frac{-2 \pm \sqrt{4-20}}{2} = -1 \pm j\frac{\sqrt{18}}{2}$.	4
(b) Main points: Transfer function (1) Poles and zeros of the transfer function (2) Plotting of poles and zeros on the s-plane (1)	4

Sample answer:
There is an error (mistyping) in the differential equation. The correct equation should be:

$$5\ddot{y} + 7\dot{y} + 9y = 3u + 2\ddot{u}$$

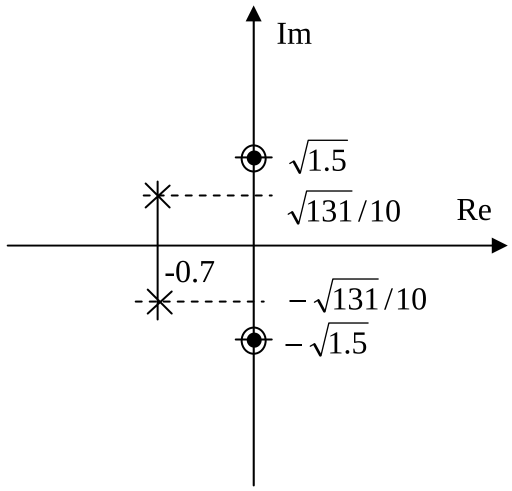
Transfer function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{3 + 2s^2}{5s^2 + 7s + 9}$$

Zeros: $3 + 2s^2 = 0$; $z_{1,2} = \pm j\sqrt{3/2}$

Poles: $5s^2 + 7s + 9 = 0$; $p_{1,2} = \dfrac{-7 \pm \sqrt{49 - 4 \times 5 \times 9}}{2 \times 5} = \dfrac{-7 \pm j\sqrt{131}}{10}$

Poles and zeros are plotted on the s-plane:



QUESTION 2

8

(a) Main points:
Block diagram & drawing a block diagram of a closed-loop control system (1)
Forward transfer function (1)
Open loop transfer function (1)
Closed-loop transfer function (1)

Sample answer:
A block (rectangle or a functional block) containing a transfer function can be used to represent the relationship between an output and an input of a dynamic system.

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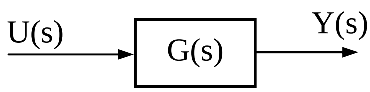
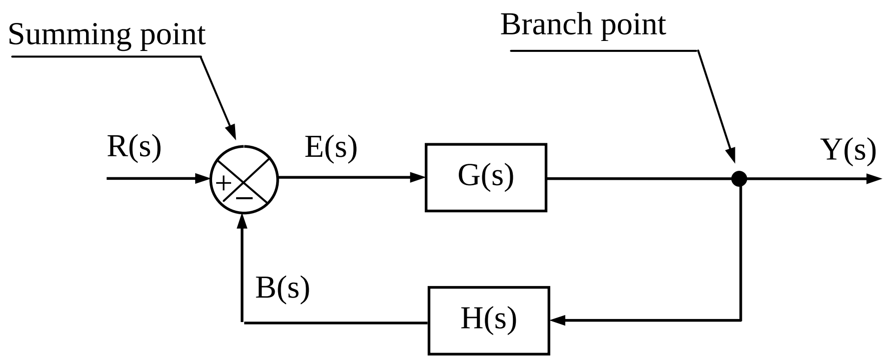


From the block we have $G(s) = \dfrac{Y(s)}{U(s)}$ or $Y(s) = G(s)U(s)$.

A number of blocks interconnected by lines become a block diagram. This is effectively a shorthand representation of a control system of a part of the system. Following is a block diagram of a closed-loop control system with summing point and branch point.



From the above block diagram, we can define the following transfer functions:

Forward transfer function = $\frac{Y(s)}{E(s)} = G(s)$ (ratio of the output to the error)

Open loop transfer function = $\frac{B(s)}{E(s)} = G(s)H(s)$ (ratio of the feedback signal to the error)

Closed-loop transfer function = $\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$ (ratio of the output to the reference signal, the closed-loop transfer function is also called feedback transfer function)

QUESTION 3

8

(a) Main points:
Definition of PID control, formulae (1)
P, I, D control actions and PID control actions (1)
Brief description of effect of the PID control in a control system (1)
Application and other stuff (1)

<u>Sample answer:</u>
PID stands for Proportional, Integral and Derivative. PID control is a control algorithm that is constructed by three individual actions: Proportional, Integral and Derivative.

4

The P control action: the value of the controller output is proportional to the actuating error:

$$u(t) = K_P e(t)$$

or in form of transfer function:

$$\frac{U(s)}{E(s)} = K_P$$

where K_P is an adjustable constant, called proportional control gain.
The I control action: the value of the controller output is changed at a rate proportional to the actuating error signal, expressed by the following formulas

$$\frac{du(t)}{dt} = K_I e(t)$$

where K_I is an adjustable constant, called integral control gain.

$$u(t) = K_I \int_0^t e(t)dt$$

or in form of transfer function

$$\frac{U(s)}{E(s)} = \frac{K_I}{s}$$

K_I can be determined by $K_I = \frac{K_P}{T_I}$ where T_I is integral time (seconds).

The D control action: the value of the controller output is change at a rate proportional to the rage of the change of the actuating error, expressed by

$$u(t) = K_D \frac{de(t)}{dt} \text{ or } u(t) = K_P T_D \frac{de(t)}{dt}$$

where K_D is an adjustable constant, called derivative control gain, T_D is derivative time (seconds).

or in form of transfer function:

$$\frac{U(s)}{E(s)} = K_D s$$

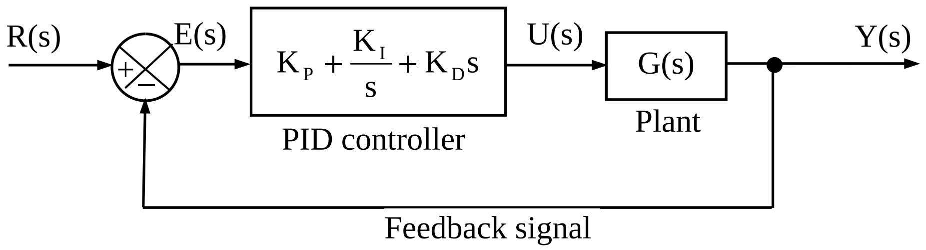
The PID control action: in the PID controller, the value of the controller output is a sum of three individual controller signals, i.e.

$$u(t) = K_P e(t) + K_I \int_0^t e(t)dt + K_D \frac{de(t)}{dt}$$

or in form of transfer function

$$\frac{U(s)}{E(s)} = K_P + K_I \frac{1}{s} + K_D s \text{ or } \frac{U(s)}{E(s)} = K_P \left(1 + \frac{1}{T_I s} + T_D s\right)$$

The following block diagram shows an example of PID control system:



(You may give brief description about the effects of P, I, D control actions on the whole control system.

(b) Main points:
Differential equation and transfer function (1)
Poles and zeros (1)
Block diagram (1)
Total feedback transfer function based on the block diagram (1) 4

<u>Sample answer:</u>
(i) The forces acting on the system are:

1. damper force $b\dot{y}$
2. inertial force $m\ddot{y}$
3. spring force ky
4. external force u

Based on the force balance, the differential equation for the mass-damper-spring system is

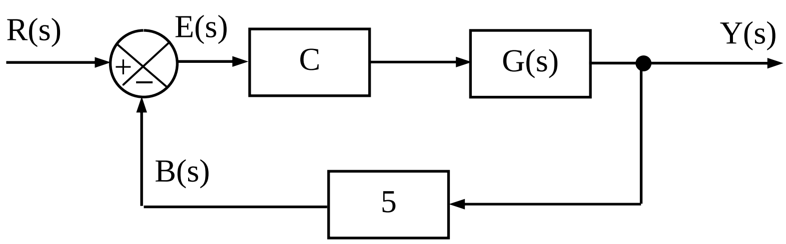
$$m\ddot{y} + b\dot{y} + ky = u$$

Transfer function is

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{ms^2 + bs + k}$$

The transfer function has no zeros, poles at $p_{1,2} = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m}$ (you may substitute values of m, k and b).

(ii) Block diagram is as follows:



where $C = \frac{K_P s + K_I}{s}$ (PI controller), $G(s) = \frac{1}{ms^2 + bs + k}$

The total feedback transfer function is

$FBTF = \frac{Y(s)}{R(s)} = \frac{CG}{1 + CG}$. Substituting the above C and G(s) and simplifying it we obtain:

$$FBTF = \frac{K_P s + K_I}{s(ms^2 + bs + k) + 5(K_P s + K_I)}$$

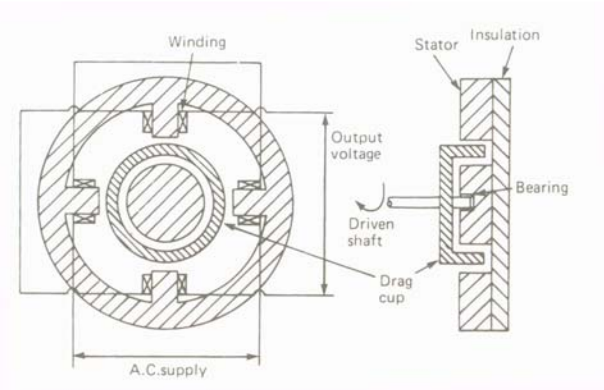
QUESTION 4

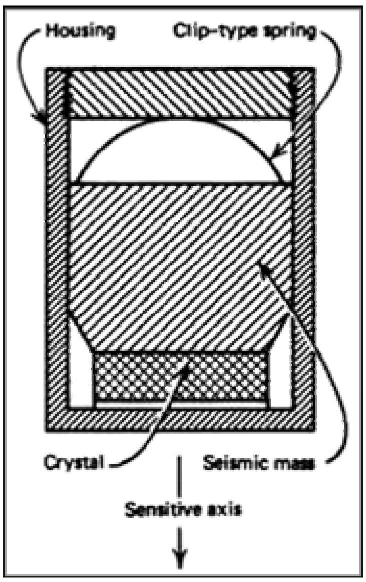
8

(a) Main points:
Structure of a differential pressure transmitter
Operating principle
Features
Applications

4

<u>Sample answer (guideline):</u>
A differential pressure transmitter is a device used to measure the differential pressure between two points and converts the differential pressure information in some type of signal, normally electrical signal. The output of a differential pressure transmitter is often in some standard industrial ranges such as 0-20mA, 4-20mA, 0-5V or 0-10V. There are many types of differential pressure

transmitters in market. You may take an example of *inductance type differential pressure transmitter with an orifice*. (b) Main points: Input pressure (current, total resistance, volt and pressure) (2) Max possible error and probable error (2) (i) $K_2 = \frac{\Delta d}{I}$; $I = \frac{\Delta d}{K_2} = \frac{60}{5} = 12\,\mu A = 12 \times 10^{-6}$ (A) $R = R_1 + R_2 = 350 + 100 = 450\ \Omega$ $V = I \times R = 12 \times 10^{-6} \times 450 = 5.4$ (mV) $K_1 = \frac{V}{\Delta P}$; $\Delta P = \frac{V}{K_1} = \frac{5.4}{2.5} = 2.16$ bar (ii) Maximum possible error = $e_1 + e_2$ = +0.75% - 0.15% = +0.5% Probable error = $\pm\sqrt{e_1^2 + e_2^2} = \pm\sqrt{0.75^2 + 0.15^2} = \pm 0.79\%$	4	
QUESTION 5 (a) Main points: Structure of a velocity measuring device (mechanic tachometer) (1) Operating principle (1) Features (1) Applications (1) <u>Sample answer:</u> <u>Electrical tachometer – drag cup</u> Structure (see figure)  Principle: The drag cup tachometer uses an aluminium cap which is rotated in a laminated iron electromagnet stator, see the above figure. The stator has two separate windings at right angles to one another. An alternating current supply is provided to one winding and eddy currents are set up in the aluminium cup. This results in an induced e.m.f (*electromotive force*) in the other stator winding which is proportional to the speed of rotation. The output voltage is measured on a voltmeter calibrated to read in units of speed. Features: The drag cup tachometer has an excitation A.C. voltage supply. There will be error if bearing wears. Applications: The drag cup tachometer can be used in cars to measure revolution of the shaft, distance and speed.	**8** 4	

(b) Main points: Overall sensitivity (with correct unit) (2) Pen movement (2) $K_1 = 0.5$ mV/mm; $K_2 = 2$ V/mV; $K_3 = 2.5$ mm/V Overall sensitivity $K = K_1 \times K_2 \times K_3 = 0.5\ (mV/mm) \times 2\ (V/mV) \times 2.5\ (mm/V) = 2.5\ (mm/mm)$ Pen movement $K = \dfrac{\Delta dp}{\Delta di} = 2.5;\ \Delta dp = 2.5 \times 2 = 5\ (mm)$	4
QUESTION 6	**8**
(a) Main points: 1. Structure of an accelerometer (1) 2. Operating principle (1) 3. Features (1) 4. Applications (1) <u>Sample answer (Piezo-electrical accelerometer):</u> The piezoelectric accelerometer is based on a property exhibited by certain crystals where a voltage is generated across the crystal when stressed. This property is also the basis for such familiar sensors as crystal phonograph cartridges and crystal microphones. For accelerometers, the principle is shown in the following figure. Here, a piezoelectric crystal is spring-loaded with a test mass in contact with the crystal. When exposed to an acceleration, the test mass stresses the crystal by a force (F = ma), resulting in a voltage generated across the crystal. A measure of this voltage is then a measure of the acceleration. The crystal per se is a very high-impedance source, and thus requires a high-input impedance, low-noise detector. Output levels are typically in the millivolt range. The natural frequency of these devices may exceed 5kHz, so that they can be used for vibration and shock measurements.  A piezoelectric accelerometer has a very high natural frequency.	4

(b) Main points: Overall sensitivity with correct unit (2) Max possible error & probable error (2) (i) K_1 = 60 pC/bar; K_2 = 5 mV/pC = 5×10^{-3} V/pC; K_3 = 2 cm/V Overall sensitivity K = $K_1 \times K_2 \times K_3$ = 60 (pC/bar) $\times$ 5 $\times10^{-3}$ (V/pC) $\times$ 2 (cm/V) = 0.6 (cm/bar) = 6 (mm/bar) (ii) Maximum possible error = $\pm e_1 \pm e_2 \pm$ e3 = $\pm 0.25\% \pm 1.25\% \pm 1.75\%$ = $\pm 3.25\%$ Probable error = $\pm\sqrt{e_1^2 + e_2^2 + e_3^2}$ = $\pm\sqrt{0.25^2 + 1.25^2 + 1.75^2}$ = $\pm 2.16\%$	4
Total marks	**48**