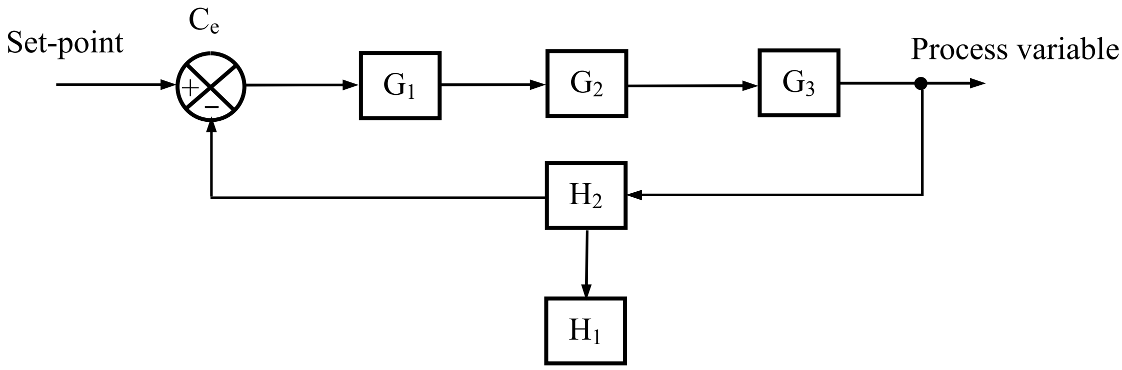
SOLUTIONS and/or MARKING SCHEME (Prepared by Hung Nguyen)

QUESTION 1

(a)
1. The general structure of a closed-loop control system is illustrated by the following block diagram:



G_3 = Plant (dynamic system)
H_1 = Indicating device (recorder, indicator, display)
H_2 = Measurement element
C_e = Comparison element
G_1 = Controller
G_2 = Final control element

Functions of components:

1. Plant (dynamic system: liquid system, pneumatic, electric system):
2. Measurement element (sensor, transducer, transmitter):
3. Indicating device (recorder, indictor, display)
4. Comparison element (potentiometer, synchro)
5. Controller (PID control, pole placement control)
6. Final control element (hydraulic valve, servo motor)

[8 Marks]

(b) (i)
1. Differential equation:

$$m\ddot{y} + b\dot{y} + ky = u$$

2. Taking Laplace transform with zero initial conditions, the transfer function is

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{ms^2 + bs + k}$$

3. Substituting m = 2 kg, b = 200 Ns/m and K = 250 N/m we have:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{2s^2 + 200s + 250}$$

No zero and poles p_1, p_2 are roots of $2s^2 + 200s + 250 = 0$

[4 Marks]

(ii) State space model:

$$\ddot{y} = -\frac{250}{2}y - \frac{200}{2}\dot{y} + \frac{1}{2}u$$

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$$

$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du}$$

where state vector: $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$; $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -125 & -100 \end{bmatrix}$; $\mathbf{B} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}$; $\mathbf{u}$ = u, $\mathbf{y}$ = y; $\mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}$ and $\mathbf{D} = 0$

[2 Marks]

QUESTION 2

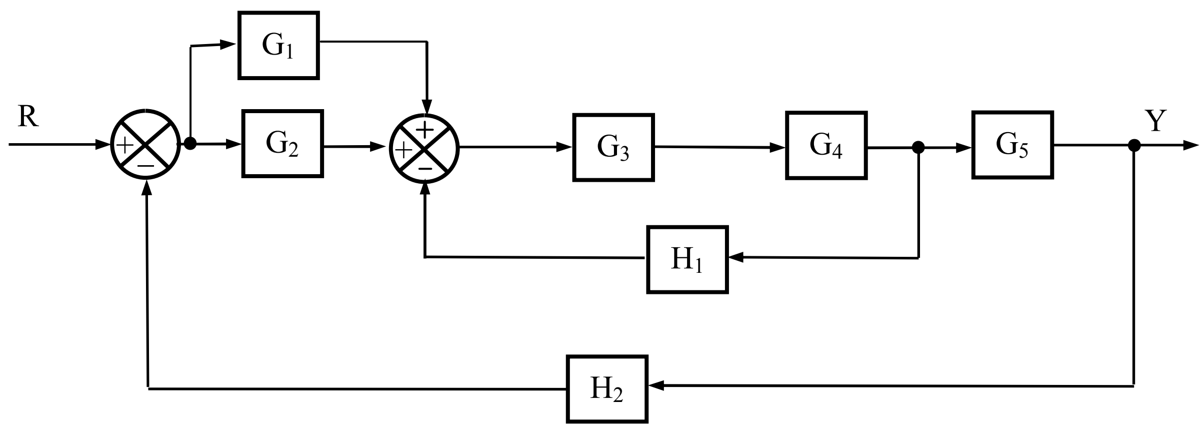
(a)

1. Concept of block diagram (functional block with input and output and transfer function, block diagram)
2. Related terms: forward path transfer function, closed-loop (feedback) transfer function

[4 Marks]

3. Reduce the block diagram using block diagram algebra:

3.1. G_1, G_2



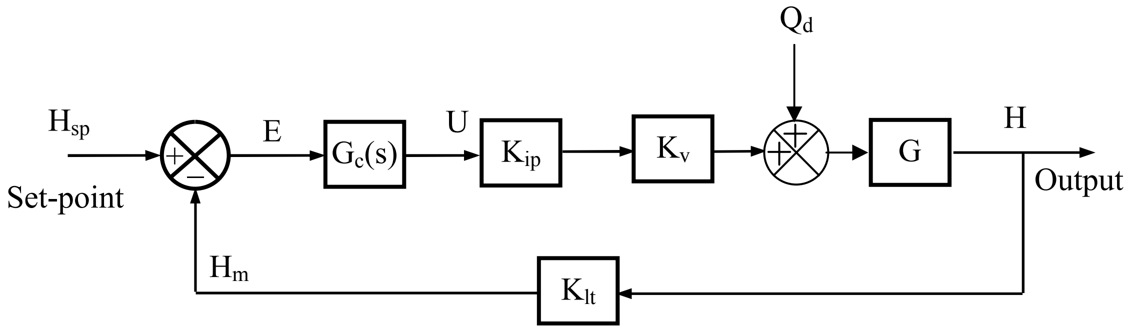
3.2. G_3, G_4, H_1

3.3 G_1, G_2, G_3, G_4, G_5, H_1

3.4 Final reduction

[4 Marks]

(b) Block diagram for the liquid level control system (annotations for signals are recommended.)



[6 Marks]

Total transfer function (with assumption $Q_d = 0$ and after reduction) from the above block diagram.

$$\frac{H}{H_{sp}} = \frac{(G_c K_{IP} K_v + Q_d) G}{1 + (G_c K_{IP} K_v + Q_d) G K_{lt}}$$

[6 Marks]

QUESTION 3

(a)

1. Concept of transfer function, definitions of poles and zeros (using formulae and sketches)
2. Poles and zeros of

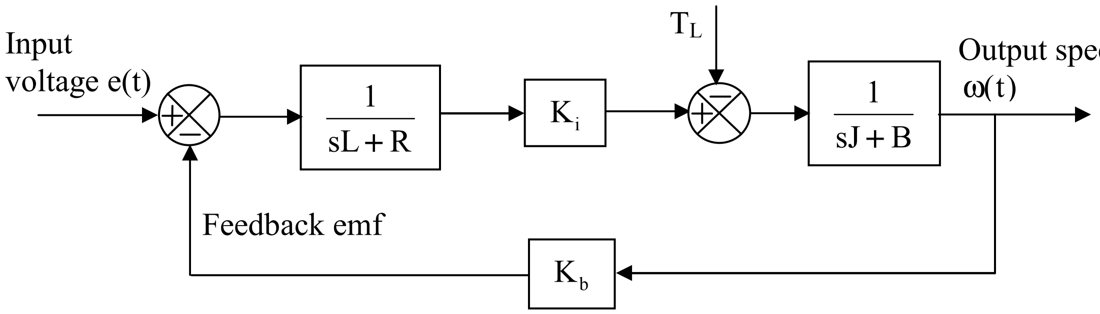
$$G(s) = \frac{2s+1}{s^2+3s+5} \quad (z_1, p_1, p_2)$$

3. Poles and zeros of

$$H(s) = \frac{s^2+2s+2}{s(s^2+3s+7)(s^2+7s+5)} \quad (z_1, z_2, p_1, p_2, p_3, p_4, p_5)$$

[6 Marks]

(b)

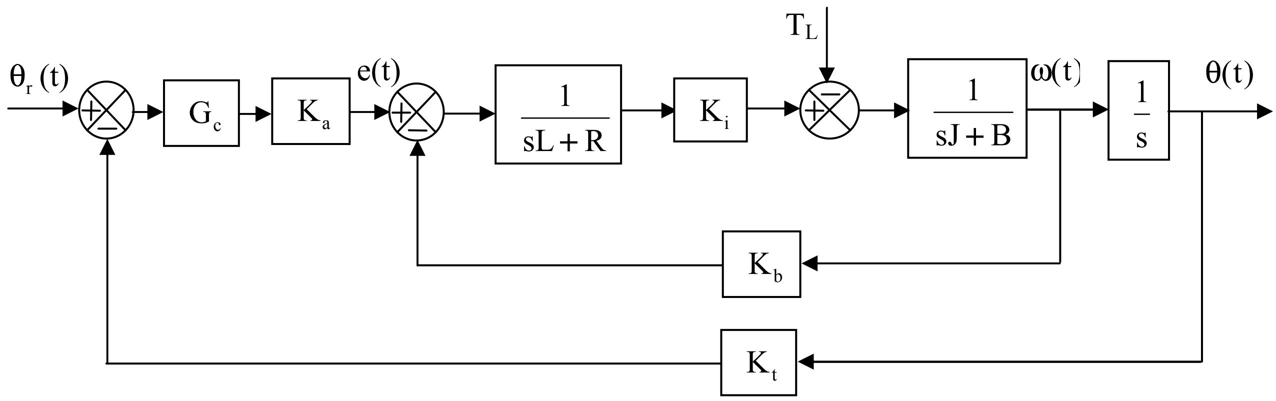


(i) Transfer function (Ω/E) of the motor system (after reduction and with assumption of T_L=0)

$$\frac{\Omega}{E} = \frac{\frac{1}{sL+R}K_i\frac{1}{sJ+B}}{1+\frac{1}{sL+R}K_i\frac{1}{sJ+B}K_b} = \frac{K_i}{(sL+R)(sJ+B)+K_iK_b}$$

[4 Marks]

(ii) Block diagram



Transfer function ($T_L = 0$)

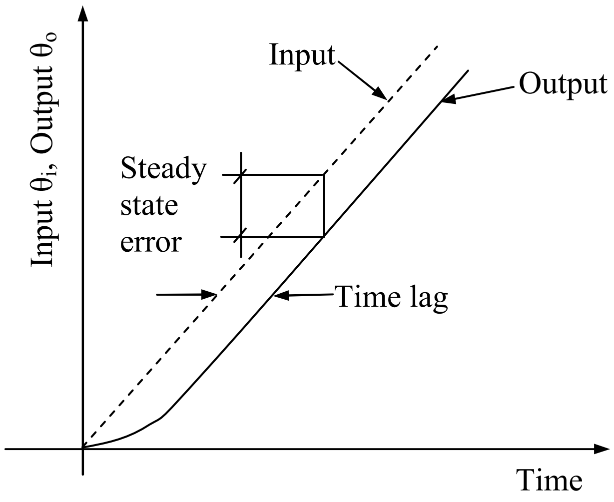
$$\frac{\Theta}{\Theta_r} = \frac{\frac{K_i}{(sL+R)(sJ+B)+K_iK_b}G_cK_a\frac{1}{s}}{1+\frac{K_i}{(sL+R)(sJ+B)+K_iK_b}G_cK_a\frac{1}{s}K_t} = \frac{K_iG_cK_a}{s(sL+R)(sJ+B)+sK_iK_b+K_iG_cK_aK_t}$$

[6 Marks]

QUESTION 4

(a)

The steady state error of a dynamic system is the difference between the output and the reference signal under the steady-state condition. The following figure illustrates the steady state error:



Concept of steady state error with a ramp input reference signal

Use formulae and block diagram of closed-loop system to illustrate the SSE.

Procedures to Calculate the Steady State Error

1. Find total feedback transfer function:

$$\text{F.B.T.F} = \frac{Y(s)}{R(s)}$$

Note that the total feedback transfer function is the ratio of the output (process variable) to the reference input (set point).

2. Apply an input signal and find the Laplace transform of error by

$$E(s) = R(s)(1 - \text{F.B.T.F})$$

3. Find the steady state error by

$$\lim_{s\to 0}[sR(s)(1 - \text{F.B.T.F})]$$

[6 Marks]

(b) Determine steady state errors of the following systems if the test signal is a unit step function.

(i) $H(s) = \dfrac{2}{4s+1}$

$$\text{F.B.T.F} = \frac{H}{1+H} = \frac{\frac{2}{4s+1}}{1+\frac{2}{4s+1}} = \frac{2}{4s+3}$$

$$\text{SSE} = \lim_{s\to 0}[sR(s)(1-\text{F.B.T.F})] = \lim_{s\to 0}\left[s\frac{1}{s}\left(1-\frac{2}{4s+3}\right)\right] = \lim_{s\to 0}\left[s\frac{1}{s}\left(1-\frac{2}{4s+3}\right)\right] = 1-\frac{2}{3} = \frac{1}{3}$$

(ii) $G(s) = \frac{1}{s^2 + 5s + 2}$

$$F.B.T.F = \frac{G}{1+G} = \frac{\frac{1}{s^2+5s+2}}{1+\frac{1}{s^2+5s+2}} = \frac{1}{s^2+5s+3}$$

$$SSE = \lim_{s\to 0}[sR(s)(1 - F.B.T.F)] = \lim_{s\to 0}\left[s\frac{1}{s}\left(1-\frac{1}{s^2+5s+3}\right)\right] = 1-\frac{1}{3} = \frac{2}{3}$$

[6 Marks]

QUESTION 5

(a) Example: U-tube manometer to measure liquid pressure
Principle: Bernoulli's equation
Structure: Flow sensor (orifice or venturi), U-tube and liquid (mercury)
Features: different ranges, advantages and disadvantages
Applications: measure liquid pressure in pipe systems

[8 Marks]

(b) (i) The maximum measurable acceleration in g:

$$a = \frac{k}{m}\Delta x = \left(\frac{2.0\times 10^3 \,N/m}{0.05kg}\right)\times(0.05m) = 2000 \text{ m/s}^2$$

(ii) The natural frequency is given by $f_N = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$$f_N = \frac{1}{2\pi}\sqrt{\frac{2.0\times 10^3 \,N/s}{0.05kg}}$$

$f_N = 31.84$ Hz

(iii) a = 500 m/s^2, the output voltage:
When the wiper in the midpoint of potentiometer, the output voltage is

$$v_o = \frac{1}{2}100\Omega\times 12V = 6V$$

a = 500 m/s^2, the displacement from the midpoint is $\Delta x = a\frac{m}{k}$ = 0.0125 (m), the output voltage is

$$v_o = 6 + \frac{50}{2}\Omega\times\frac{12}{100}V = 9 \text{ V}$$

[6 Marks]

QUESTION 6

(a) Example: A type of control valve (diaphragm valve)
1. Principle
2. Structure
3. Features or characteristics
4. Applications

[8 Marks]

(b)
Overall sensitivity: $K = K_1K_2K_3$ = 50 (pC/bar)$\times 5\times 10^{-3}$ (V/pC)$\times$1 (cm/V) = 25×10^{-2} (cm/bar)
Trace deflection: $\Delta d = K\times P = 25\times 10^{-2}$ (cm/bar) $\times$ 30 bar = 7.5 cm = 75 mm
Maximum possible error $e = e_1 + e_2 + e_3 = \pm 0.5\% \pm 0.75\% \pm 1.25\% = \pm 2.5\%$
Probable error = $\pm\sqrt{e_1^2 + e_2^2 + e_3^2} = \pm\sqrt{0.5^2 + 0.75^2 + 1.25^2} = \pm 1.54\%$

[4 Marks]

SOLUTIONS and/or MARKING SCHEME (Prepared by Hung Nguyen)

QUESTION 1

(a) Concepts of transfer function, poles and zeros:

The transfer function for a dynamic system is defined as ratio of Laplace transform of the output of the system to the corresponding Laplace transform of the input with zero conditions. In general, if a dynamic system is represented by the following differential equation:

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 \dot{y} + a_0 y = b_m u^{(m)} + b_{m-1} u^{(m-1)} + \cdots + b_1 \dot{u} + b_0 u$$

then the transfer function is

$$H(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0} = \frac{N(s)}{D(s)}$$

Poles of the above transfer function are defined as roots of the following equation:

$D(s) = 0$, i.e. $a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0 = 0$

(Significance of poles: At poles transfer function tends to infinity. Poles determine the dynamic performance of a dynamic system.)

Zeros of the above transfer function are defined as roots of the following equation:

$N(s) = 0$, i.e. $b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0 = 0$

(Significance of zeros: At zeros, the transfer function is equal to zero.)

Taking Laplace transform of the equation with zero initial condition yields

$$as^2 Y(s) + bsY(s) + cY(s) = dU(s)$$

Applying the above definition of T.F. gives

$$G(s) = \frac{Y(s)}{U(s)} = \frac{d}{as^2 + bs + c}$$

Inserting values of a, b, c and d yields

$$G(s) = \frac{Y(s)}{U(s)} = \frac{2}{s^2 + 3s + 10}$$

This transfer function has no zeros and has poles at $p_{1,2} = \frac{-3 \pm \sqrt{9 - 4 \times 10}}{2} = -1.5 \pm j\frac{\sqrt{31}}{2}$

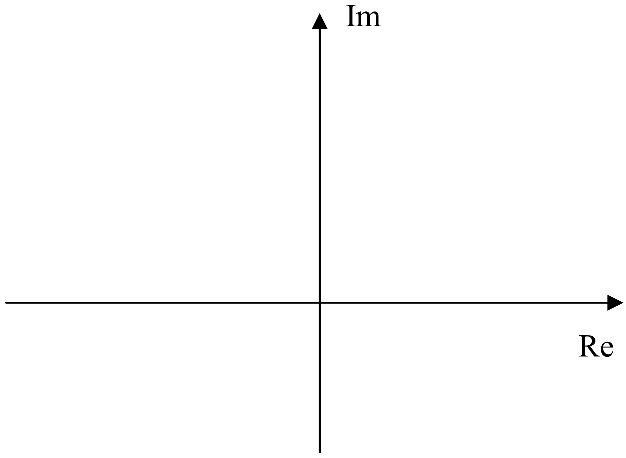
(b) (i) Zero: z = -1, $p_{1,2} = \frac{-2 \pm \sqrt{4 - 20}}{2} = -1 \pm 2j$

(ii) Zero: z = -1/2, Poles: $p_{1,2} = 0$, $p_{3,4} = \frac{-1 \pm \sqrt{1 - 4 \times 3}}{2} = -0.5 \pm j\frac{\sqrt{11}}{2}$

(iii) No zero, poles: $p_{1,2} = -\omega_n \zeta \pm \sqrt{(\omega_n \zeta)^2 - \omega_n^2} = -\omega_n \zeta \pm \omega_n \sqrt{\zeta^2 - 1}$

[6 Marks]

(Plot poles and zeros in the s-plane)



[6 Marks]

QUESTION 2

(a) State functions of

(1) G_2 Plant: the dynamic system with process variables to be controlled

(2) H Measurement Element: to measure process variables

(3) Comparison Element: to compare the set-point value and the measured variable and generate actuating error signal

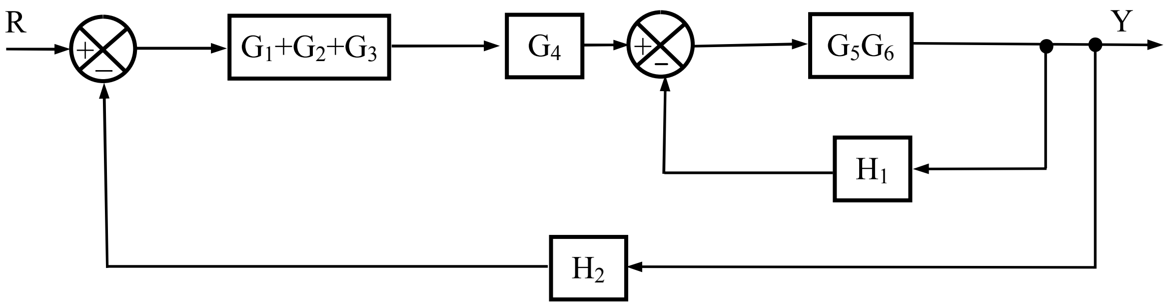
(4) C Controller: to compute control signals $u_c(t)$

(5) G_1 Final Control Element (valve or servo): to amplify and/or convert signals and actuate the plant in order to make the process variables to reach the desired values

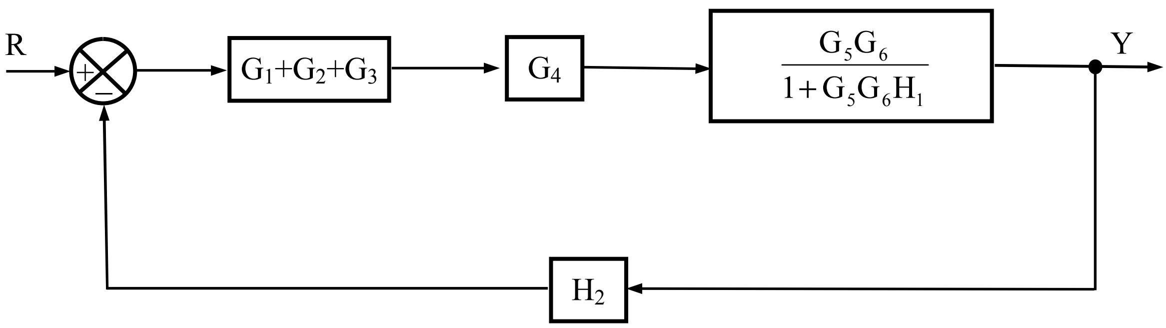
[6 Marks]

(b)

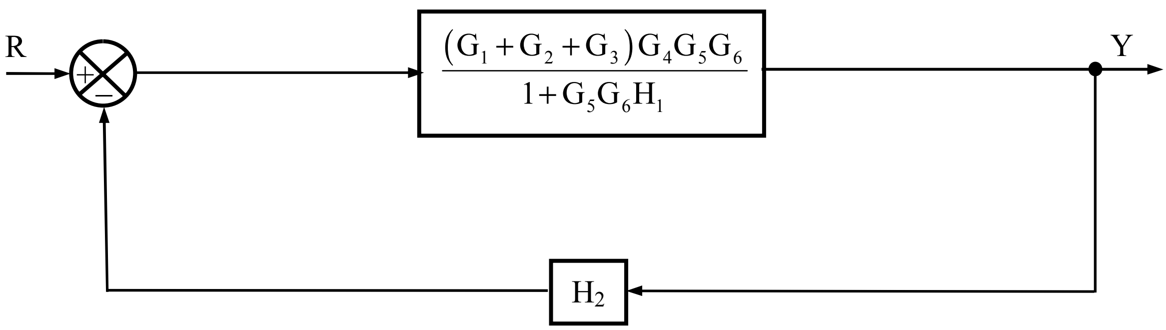
Step 1:



Step 2



Step 3



Step 4 (including simplification of the total transfer function)

$$R \longrightarrow \boxed{\dfrac{\dfrac{(G_1+G_2+G_3)G_4G_5G_6}{1+G_5G_6H_1}}{1+\dfrac{(G_1+G_2+G_3)G_4G_5G_6}{1+G_5G_6H_1}H_2}} \longrightarrow Y$$

Final result

$$R \longrightarrow \boxed{\dfrac{(G_1+G_2+G_3)G_4G_5G_6}{1+G_5G_6H_1+(G_1+G_2+G_3)G_4G_5G_6H_2}} \longrightarrow Y$$

[4 Marks]

QUESTION 3

(a)

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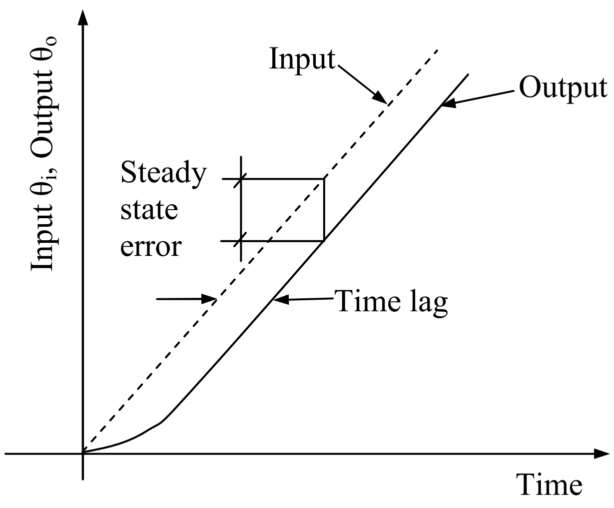

Concept of steady state error with a ramp input reference signal

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1. Find total feedback transfer function:

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Note that the total feedback transfer function is the ratio of the output (process variable) to the reference input (set point).

2. Apply an input signal and find the Laplace transform of error by

$$E(s) = R(s)(1 - F.B.T.F)$$

3. Find the steady state error by

$$\lim_{s \to 0}[sR(s)(1 - F.B.T.F)]$$

[6 Marks]

(b) Determine steady state errors of the following systems if the test signal is a unit step function.

(i) $H(s) = \frac{3}{5s+1}$

$$F.B.T.F = \frac{H}{1+H} = \frac{\frac{3}{5s+1}}{1+\frac{3}{5s+1}} = \frac{3}{5s+4}$$

$$SSE = \lim_{s \to 0}[sR(s)(1 - F.B.T.F)] = \lim_{s \to 0}\left[s\frac{1}{s}\left(1 - \frac{3}{5s+4}\right)\right] = \lim_{s \to 0}\left[s\frac{1}{s}\left(1 - \frac{3}{5s+4}\right)\right] = 1 - \frac{3}{4} = \frac{1}{4}$$

(ii) $G(s) = \frac{2}{2s^2+3s+5}$

$$F.B.T.F = \frac{G}{1+G} = \frac{\frac{2}{2s^2+3s+5}}{1+\frac{2}{2s^2+3s+5}} = \frac{2}{2s^2+3s+7}$$

$$SSE = \lim_{s \to 0}[sR(s)(1 - F.B.T.F)] = \lim_{s \to 0}\left[s\frac{1}{s}\left(1 - \frac{2}{2s^2+3s+7}\right)\right] = 1 - \frac{2}{7} = \frac{5}{7}$$

[6 Marks]

QUESTION 4

(a) A differential pressure transmitter consists of a flow sensor (an orifice or venturi), a type of differential pressure transducer (l.v.d.t. or resistance type) and an indicator based on the Bernoulli's equation. The answer includes operating principle, simple structure, features and applications of the described d/p transmitter.

[8 Marks]

(b) Answer: v_f = 20 m/s, d = 40 m.

[2 Marks]

(c) Ship speed in m/s v = 15 knots × 1852 m / 3600 (s) = 7.72 m/s

f_t = 400 kHz = 400×10^3 Hz; c = 1,500 m/s, and $\theta = 60^o$

Doppler shift (single-element transducer): $f_d = \frac{2vf_t}{c}\cos\theta$ (inserting f_t = 400 kHz = 400×10^3 Hz; c = 1,500 m/s, and $\theta = 60^o$)

[2 Marks]

QUESTION 5

(a) Control valve is an actuator. One type is pneumatically actuated control valve that is widely used in piping systems. The answer includes operating principle, simple structure, features/characteristics and applications.

(b) We have $\rho_w = 1000$ kg/m^3; $\rho_m = 13.6 \times 10^3$ kg/m^3; h = 0.5 m

Differential pressure: $\Delta P = (\rho_m - \rho_w)gh$

[2 Marks]

QUESTION 6

(a) Applying the Kirchhoff's voltage law to the network, we have

$$v_R + v_L + v_C - e_i(t) = 0$$

where

$$v_R = Ri$$

$$v_L = L\frac{di}{dt}$$

$$v_C = \frac{1}{C}\int_0^t idt$$

$$v_C = e_o(t) = \frac{1}{C}\int_0^t idt$$

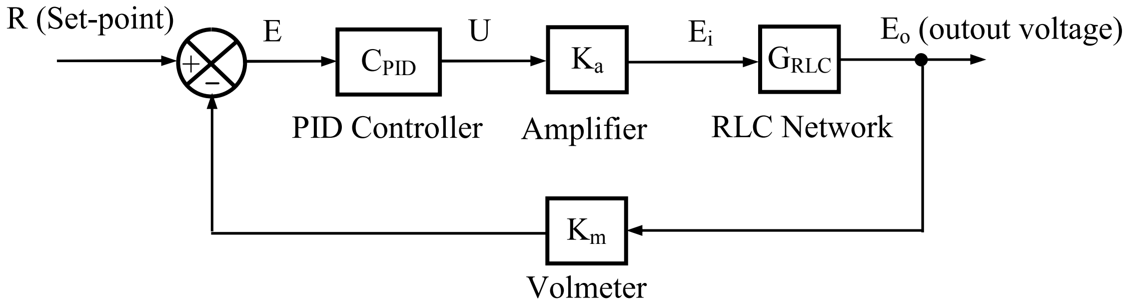
The differential equation relating $e_o(t)$ and $e_i(t)$ is

$$LC\ddot{e}_o + RC\dot{e}_o + e_o = e_i$$

Derive transfer function, insert values of L, C and R and find poles and zeros.

[8 Marks]

(b) Block diagram



where $C_{PID} = K_p + K_i\frac{1}{s} + K_d s = \frac{K_d s^2 + K_p s + K_i}{s}$

$$G_{RLC} = \frac{1}{LCs^2 + RCs + 1}$$

Total feedback transfer function:

$$\text{F.B.T.F.} = \frac{E_o}{R} = \frac{C_{PID}K_aG_{RLC}}{1 + C_{PID}K_aG_{RLC}K_m}$$ (insert C_{PID} and G_{RLC} and simplify)

[8 Marks]