SOLUTIONS TO CLASS TEST 1

QUESTION 1

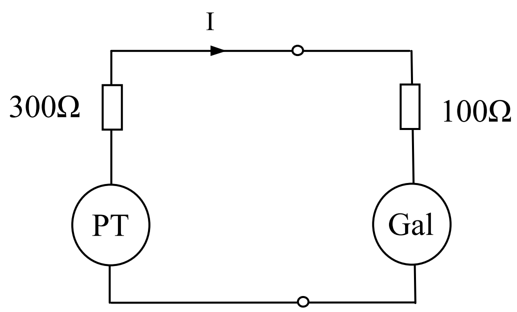
(**a**) State pressure, unit/s for pressure, absolute pressure, gauge pressure and atmospheric pressure and list methods of measuring pressure.

[4 marks]

SOLUTION

1. Pressure
2. Units for pressure
3. Absolute, gauge, and atmospheric pressures
4. List of pressure measuring methods

(**b**) A pressure piezo-electric transducer has a sensitivity of 4.0 mV/bar and an output impedance of $300\,\Omega$. If this transducer is connected to a galvanometer of resistance $100\,\Omega$ having a sensitivity of 5 mm/μA as shown in the following figure, calculate the pressure being measured if the galvanometer spot deflects 80 mm on the ultra-violet-sensitive paper.



[4 marks]

SOLUTION

$K_2 = \frac{\Delta d}{I}$, $I = \frac{\Delta d}{K_2} = 80/5 = 16\mu A = 16\times10^{-6} A$

$R = R_1 + R_2 = 300 + 100 = 400$ Ohms

$V = IR = 400\times16\times10^{-6} A = 6400\times10^{-6} V = 6.4$ mV

$K_1 = \frac{V}{\Delta P}$ $\Delta P = 6.4/4.0 =$ **<u>1.6 bars</u>**

QUESTION 2

(**a**) What is the Doppler effect? State the principle of the ship's Doppler speed log.

[1+3 marks]

SOLUTION

1. Concept of Doppler effect
2. Transmitter and receiver onboard the ship
3. Formula for Doppler shift and ship's speed
4. Methods to reduce errors

(**b**) Calculate the frequencies of cyclic waves that have periodic times (period) of (i) 25 s, (ii) 450 ms, and (iii) 200 µs, and the periodic times of square waves that have frequencies of (iv) 5 Hz, (v) 250 MHz, and (vi) 3.0 GHz.

[2 marks]

SOLUTION

(i) $f = 1/T = 1/25 = 0.04$ Hz

(ii) $f = 1/T = 1/450\times10^{-3} = 2.2$ Hz

(iii) $f = 1/T = 1/200\times10^{-6} = 5\times10^{3}$ Hz = 5 kHz

(iv) $T = 1/f = 1/5 = 200$ms

(v) $T = 1/f = 1/250\times10^{6} = 4\times10^{-9} = 4$ ns

(vi) $T = 1/f = 1/3.0\times10^{9} = 0.33\times10^{-9} = 0.33$ ns =

(**c**) A ship has a two-element Doppler speed log with the ultrasonic beams at each angle of 60° to the horizontal as illustrated in the following figure. The ultrasonic wave has the frequency of 400 kHz, and the velocity of propagation of 1500 m/s in seawater. The ship speed is 18 knots (1NM = 1852 m). Calculate the Doppler shift.

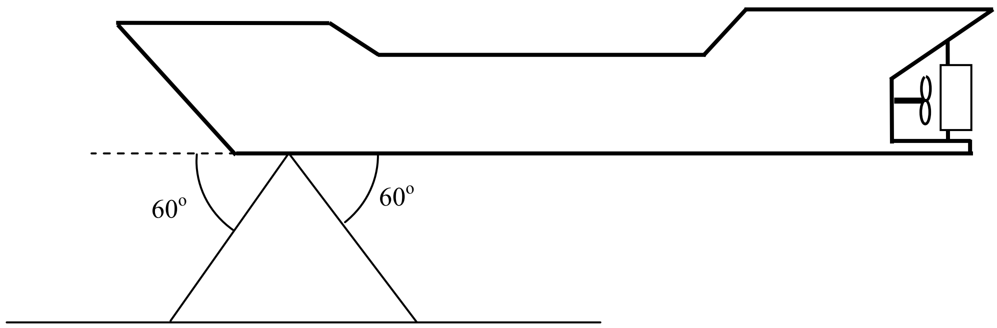


Illustration of a two-element Doppler speed log

[2 marks]

SOLUTION

$v = 18\times\frac{1852}{3600} = 9.26$ m/s

Doppler shift $f_d = \frac{2vf_f}{C}(+\cos\theta + \cos\theta') = \frac{2vf_f}{C} = \frac{2\times9.26\times400\times10^3}{1500} = 4.94\times10^3$ Hz

$= 4.94$ kHz

<u>QUESTION 3</u>

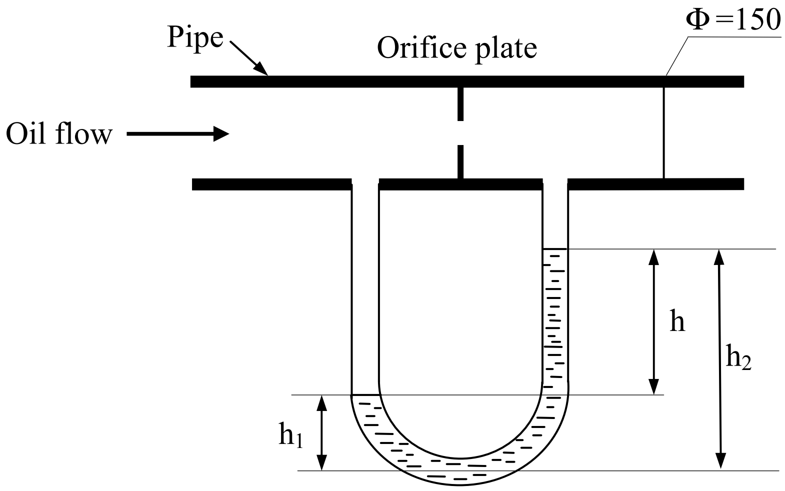
(**a**) Describe the general block diagram of a measuring system and state the functions of each block.

[4 marks]

SOLUTION

1. Structure of a measuring system
2. Functions of sensing element (transducer)
3. Functions of signal conditioning block
4. Functions of display (recorder, indicator) block

(**b**) A U-tube mercury manometer and an orifice plate are applied in measurement of weight of oil flowing through a pipe in an offshore production system as shown in the following figure. The pipe has the internal diameter of 150 mm, and a constant k of 1.25. The difference in the mercury levels in the two legs is 50 mm. The oil density is 850 kg/m^3, and mercury density is 13.56×10^3 kg/m^3.



Calculate:
(i) Differential pressure;
(ii) Volumetric flow of oil in the pipe;
(iii) Weight of oil flowing through the pipe for one day (g = 9.8 m/s^2).

[4 marks]

SOLUTION

$h = 50\text{mm} = 50\times10^{-3} = 0.05\ (\text{m})$

$d = 150\text{mm} = 150\times10^{-3} = 0.15\ (\text{m})$

(i) The differential pressure is

$\Delta P = (\rho-\rho_o)gh$

$= (13.56\text{-}0.850)\times10^3\,(\text{kg/m}^3)\times9.81(\text{m/s}^2)\times 50\times10^{-3}\,(\text{m})$

$= 12.71\times0.981\times0.5\times10^3\ \text{N/m}^2 = 6.234255\times10^3\ \text{Pa} = 6.23\ \text{kPa}$

(ii) Volumetric flow

$Q = VA$

$$V = k\sqrt{\frac{\Delta P}{\rho_o}} = 1.25\sqrt{\frac{6.23\times10^3}{850}} = 3.384\ \text{m/s}$$

$$A = \left(\frac{d}{2}\right)^2\pi = \left(7.5\times10^{-2}\right)^2\times3.14 = 0.00785\text{m}^2 = 176.625\times10^{-4} = \text{m}^2$$

$Q = 3.384\text{m/s}\times176.625\times10^{-4}\ \text{m}^2 = 0.0598\ \text{m}^3/\text{s}$

$W = Q\rho_o = 0.0598\ \text{m}^3/\text{s}\times850\ \text{kg/m}^3 = 50.83\ \text{kg/s}$

$W_{(day)} = W\times3600\times24 = 50.83\times3.6\times24\times10^3$

$= 4391.712$ tons

QUESTION 4

(**a**) Describe one type of flowmeter (including simple structure, operating principle, range of input and output and features) you have known the best.

[4 marks]

SOLUTION

1. Structure of orifice
2. Operating principle
3. Range of input and output, application
4. Features

(b) Determine the time constant (T) and the static sensitivity (K) of the systems whose dynamic performances are described by the following differential equations:

(i) $80\dot{y} + 5y = 2.5\times10^{-5}u$

for a thermocouple in a protective sheath, where y is the output voltage (V) and u is the input temperature (^{o}C).

(ii) $20\dot{r} + 2.5r = 17.5\times10^{-2}\delta$

for a ship, where r is the output yaw rate in rad/s and δ is the input rudder in rad.

(iii) $m\ddot{y} + b\dot{y} + ky = u$

for a mass-spring-damper system, where u is the input force (N), y is the output displacement of mass (m). The system has a mass m = 200 kg, a spring constant $k = 0.6\times10^{3}$ N/m and damper constant b = 100 Ns/m. Calculate the un-damped natural frequency and damping ratio.

[4 marks]

SOLUTION

(i) Standard form $\frac{80}{5}\dot{y} + y = \frac{2.5\times10^{-5}}{5}u$,

time constant = 16 seconds, sensitivity = 5×10^{-6} V/^{o}C

(ii) Standard form $\frac{20}{2.5}\dot{r} + r = \frac{17.5\times10^{-2}}{2.5}\delta$

time constant = 8 seconds, sensitivity = 7×10^{-2} rad/s/rad

(iii) Standard form: $\frac{m}{k}\ddot{y} + \frac{b}{k}\dot{y} + y = \frac{1}{k}u$,

natural frequency and damping ratio:

$$\frac{1}{\omega_n^2} = \frac{m}{k} \quad \omega_n = \sqrt{\frac{k}{m}} = 1.732 \text{ (rad/sec)}$$

$$\frac{2\xi}{\omega_n} = \frac{b}{k} \quad \xi = \frac{1}{2}\frac{b}{k}\omega_n = \frac{1}{2}\frac{b}{k}\sqrt{\frac{k}{m}} = \frac{0.5b}{\sqrt{km}} = 0.1443$$

sensitivity: $K = \frac{1}{k} = 1.66\times10^{-3}$ (m/N)

QUESTION 5

(**a**) Describe a type of displacement transducer (simple structure, operating principle, range of displacement) you have known the best.

[4 marks]

SOLUTION
1. Simple structure
2. Operating principle
3. Range of displacement
4. Features

(**b**) Before leaving the ground an airplane travelling with constant acceleration makes a run on the runway of 2400 m in 20 seconds. Find: a) acceleration; b) speed at which it leaves the ground, c) distance travelled during the first and twentieth seconds.

[4 marks]

SOLUTION

$d = v_o t + \frac{at^2}{2}$, $v_o = 0$, $a = \frac{2d}{t^2} = \frac{2400 \times 2}{20 \times 20} = \underline{12\ m/s^2}$

$v_f = a*t = 12*20 = \underline{240\ m/s}$

$d_{1\text{-}2} = \frac{at^2}{2} = 12/2 = \underline{6\ m}$

$d_{20} = \frac{at^2}{2} = \frac{12 \times 20^2}{2}$

$d_{19} = \frac{at^2}{2} = \frac{12 \times 19^2}{2}$

$d_{19\text{-}20} = d_{20} - d_{19} = \frac{12 \times \left(20^2 - 19^2\right)}{2} = \underline{234\ m}$

QUESTION 6

(**a**) State the following terms: static sensitivity, accuracy and precision, possible and probable (root-sum-square) errors and linearity. Take examples to illustrate your answers.

[4 marks]

SOLUTION
1. Static sensitivity
2. Accuracy and precision
3. Possible and probable (root-sum-square) erros
4. Linearity

Example for each

(**b**) A pressure measuring system involves the use of a pizeo-electric transducer, a charge amplifier and a recorder. The individual sensitivity is $K_1 = 50$ pC/bar, $K_2 = 6.0$ mV/pC, and $K_3 = 2$ cm/V. The maximum individual errors are +0.15%, +1.25% and +1.5%, respectively.

(i) Calculate the overall sensitivity;
(ii) Calculate the maximum possible system error;
(iii) Calculate probable (root-sum-square) error.
(iv) If the input pressure is 2.0 bars, calculate the pen movement of recorder.

[4 marks]

SOLUTION

(i) $K = K_1K_2K_3 = 50\ (\text{pC/bar}) \times 6.0 \times 10^{-3}\ (\text{V/pC}) \times 2\ (\text{cm/V}) = \underline{0.6\ \text{cm/bar}}$

(ii) Maximum error = +0.15% + 1.25% + 1.5% = <u>2.9 %</u>

(iii) Probable error $= +\sqrt{0.15^2 + 1.25^2 + 1.5^2} = +\underline{1.958\%}$

(iv) $\Delta d = \Delta P \times K = 2.0\ (\text{bars}) \times 0.6\ (\text{cm/bar}) = \underline{1.2\ \text{cm}}$