Tutorial 4

Simulation of Second-order Systems

Aim

- To solve second-order ODEs
- To use simulation programs to learn dynamics of second-order systems

Learning Objectives

- Use Simulink to simulate dynamic systems (2nd-order systems)
- Create subsystem
- Mask subsystem

Background

A mass-spring-damper system is shown in **Fig. 1** in which the input force u is in N, m is mass [kg], k is spring stiffness, and b is viscous damping coefficient, y is the displacement of the mass. Use these numerical values: m = 20 kg, k = 2.0 N/m, and b = 4.0 N/(m/s) and u = -8/+8 N.

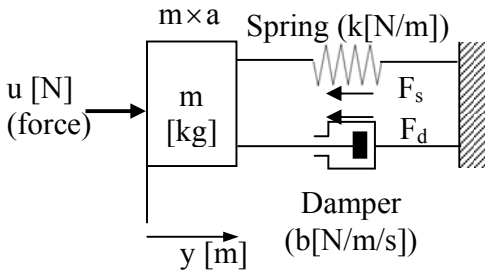

Figure 1 Mass-spring-damper system (typical mechanical system)

The mass-spring-damper is expressed by a second-order ODE below:

$$m\ddot{y} + b\dot{y} + ky = u \qquad (1)$$

Equation (1) can be rewritten as follows:

$$\ddot{y} = \frac{1}{m}\left(-ky - b\dot{y} + u\right) \qquad (2)$$

A block diagram algorithm for (2) is shown in **Fig. 2**.

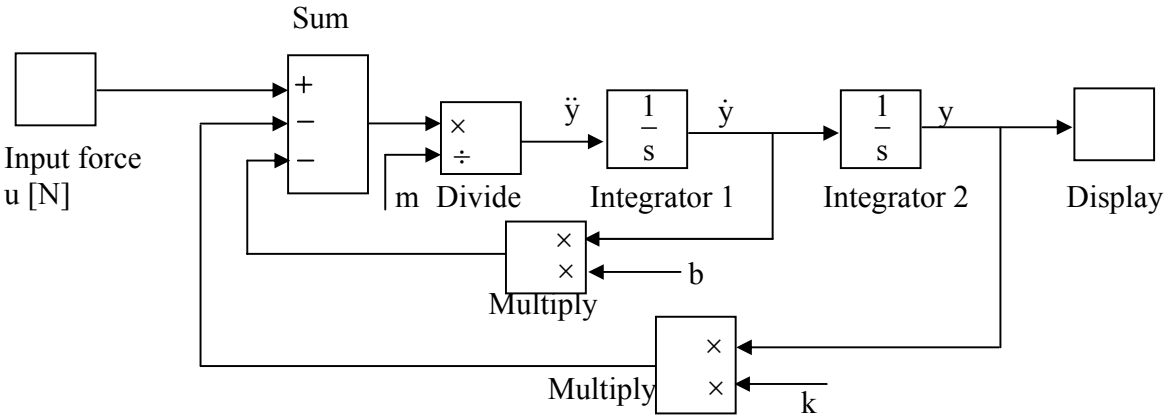

Figure 2 Block diagram of a second-order differential equation using integrators

Hands-on Exercise

Programming

- Open a Simulink model and save as “SecondOrderSysSim_Tute_04_01.mdl”.
- Add necessary block and wire them as in the following figure.

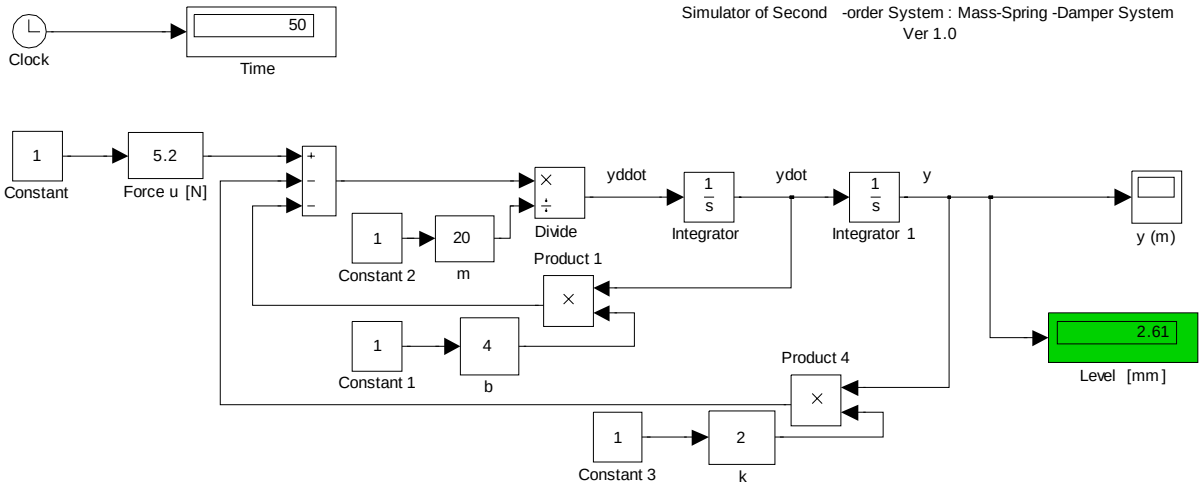

Figure 3 Simulink model for the second-order system

System Parameters

U = 5.2 [N], m = 20 [kg], b = 4 [N/(m/s)] and k = 2 [N/m]

Initial Conditions

- Double click Integrator, ydot(0) = 0
- Double click Integrator 1, y(0) = 0.
- Save the program.

Simulation Parameters

- Fixed step: 0.1
- Run the simulation program.

The result (**Fig. 4**) is obtained.

You can test functionality by changing values of input force u.

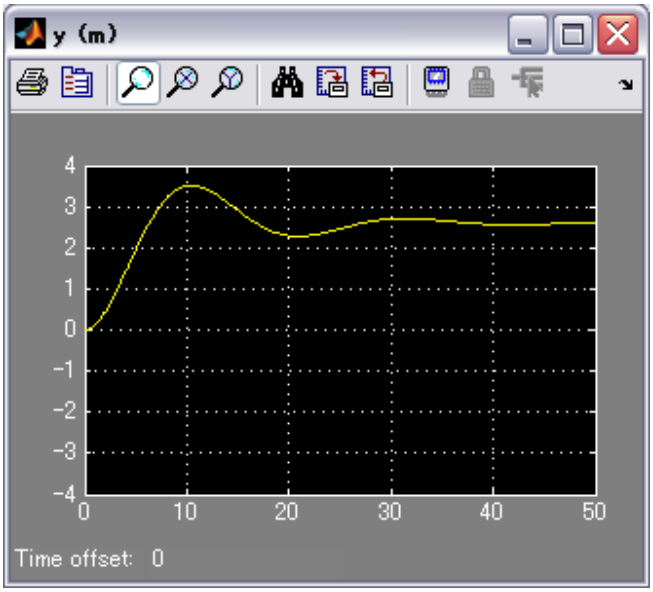

Figure 4 Scope of y [m]

- Save the program (if it is not).

Create a Subsystem

- Save the model as “SecondOrderSysSimTutorial_04_02.mdl”
- Modify the model as follows

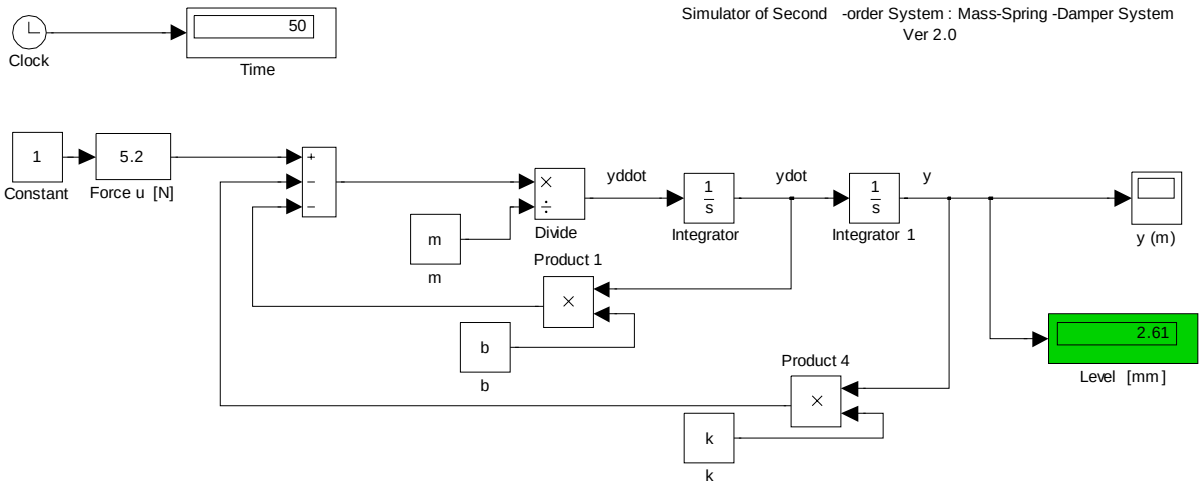

Figure 5 Simulink model with variables

- Save the model.
- Try to run the model. The following error message appears:

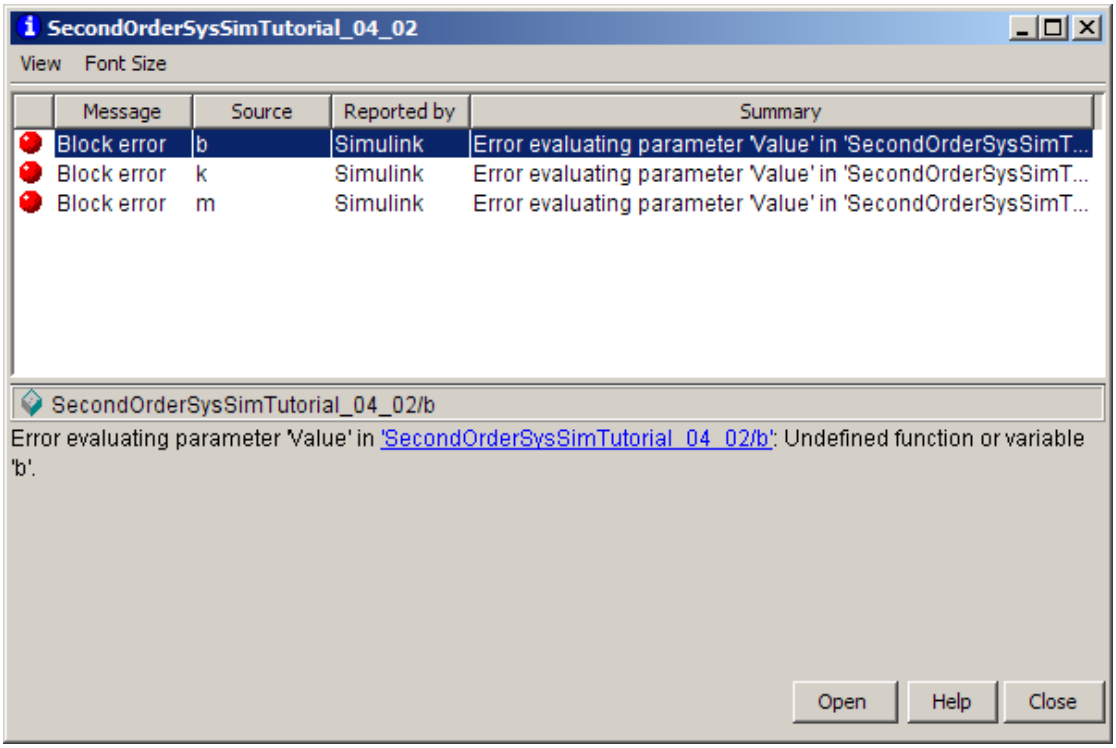

Figure 6 Error message (undefined variables b, k and m)

In order to debug the program (correct errors), do the following:

- Type the following command in Command Window:

```
>>m=20
>>b=4
>>k=2
```

- Run the Simulink model. The model should give the same result.
- Select blocks as in the figure below.

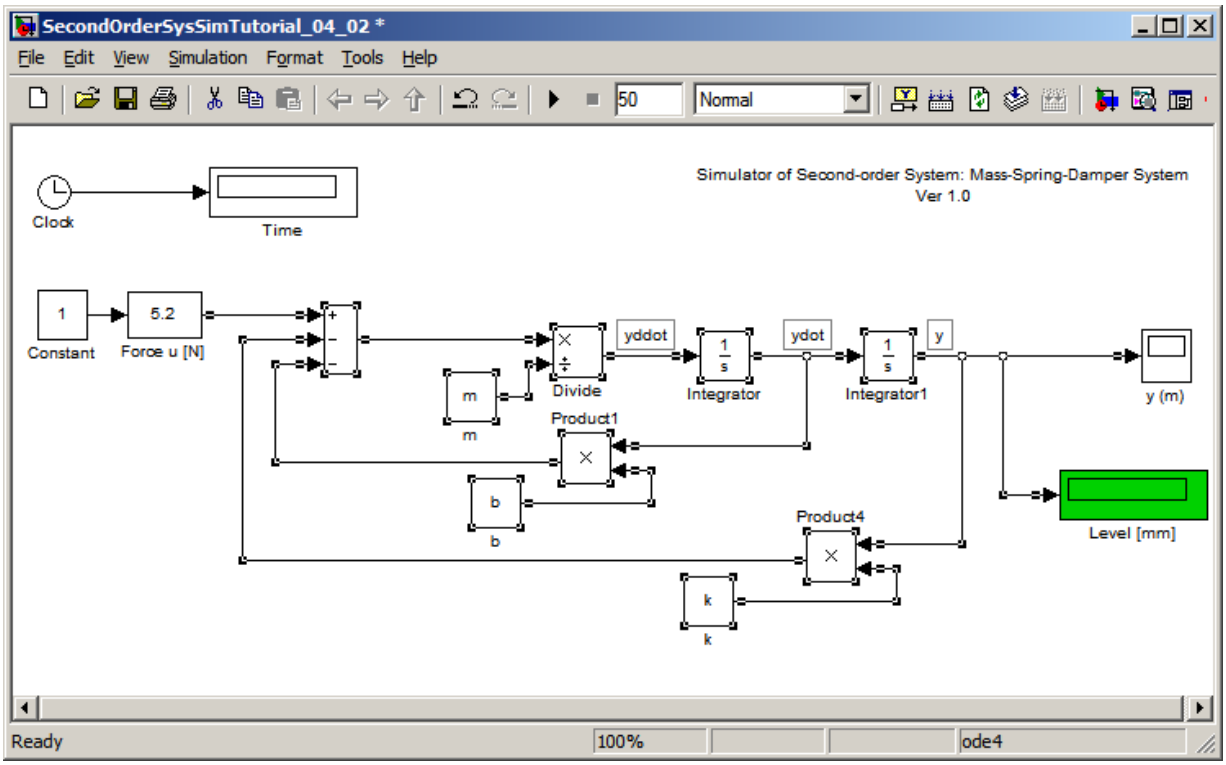

Figure 7 Selected blocks

- Edit menu > Create Subsystem (or right-click the selected blocks, Create Subsystem)

The following subsystem is obtained:

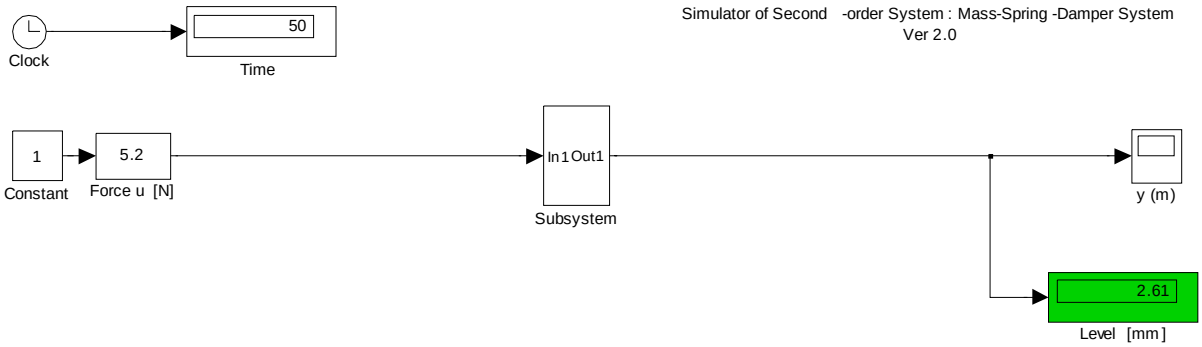

Figure 8 Subsystem

- Change Subsystem to Mass-Spring-Damper System.
- Double click the Mass-Spring-Damper System and edit In1 and Out1 as follows

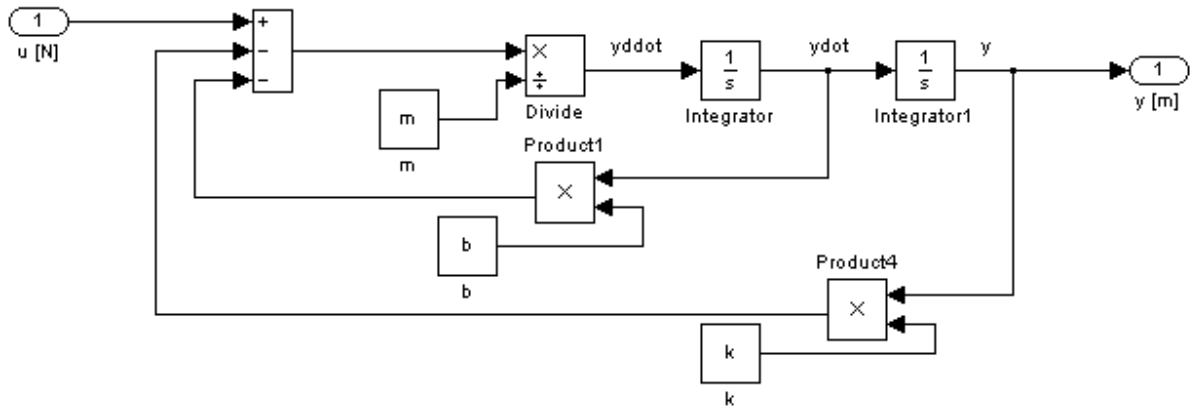

Figure 9 Subsystem "Mass-Spring-Damper System"

- Close the Subsystem. The Simulink model looks like…

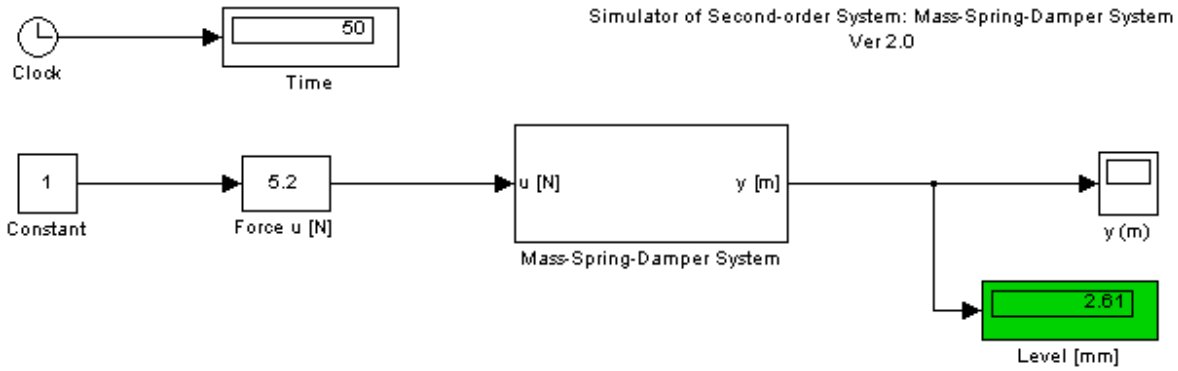

Figure 10 Simulink model with subsystem block "Mass-Spring-Damper System"

- Save the model.
- Run the model (the model will be running if m, b and k are still in Workspace). The same result should be obtained.

Mask a Subsystem

- Select Mass-Spring-Damper System block
- Edit menu > Mask Subsystem (or Right-click the Mass-Spring-Damper System block)
- Icon tab (the picture file “msdsys.PNG” is in the same folder) (Right click > Edit Mask or Look Under Mask)

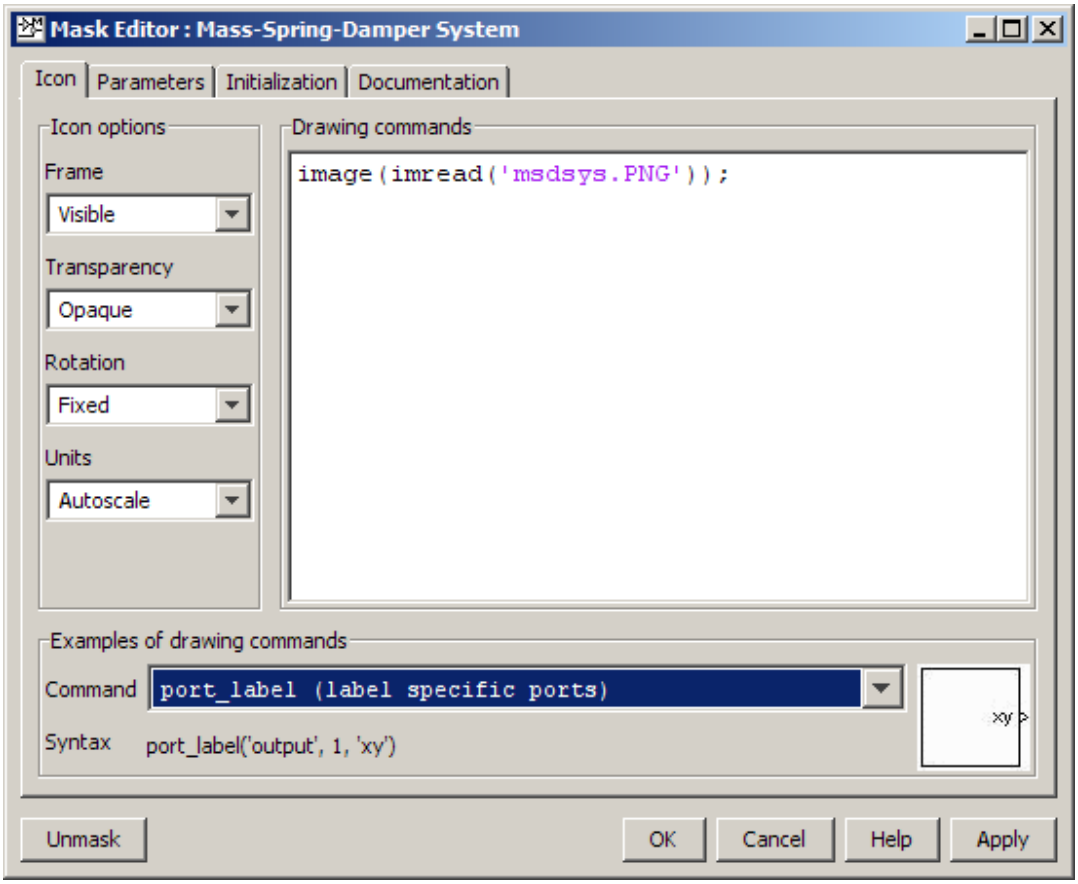

Figure 11 Icon tab of the Mask Editor

- Parameters tab: see **Fig. 12**.
- Initialization tab: see **Fig. 13**.
- Documentation tab: see **Fig. 14**.

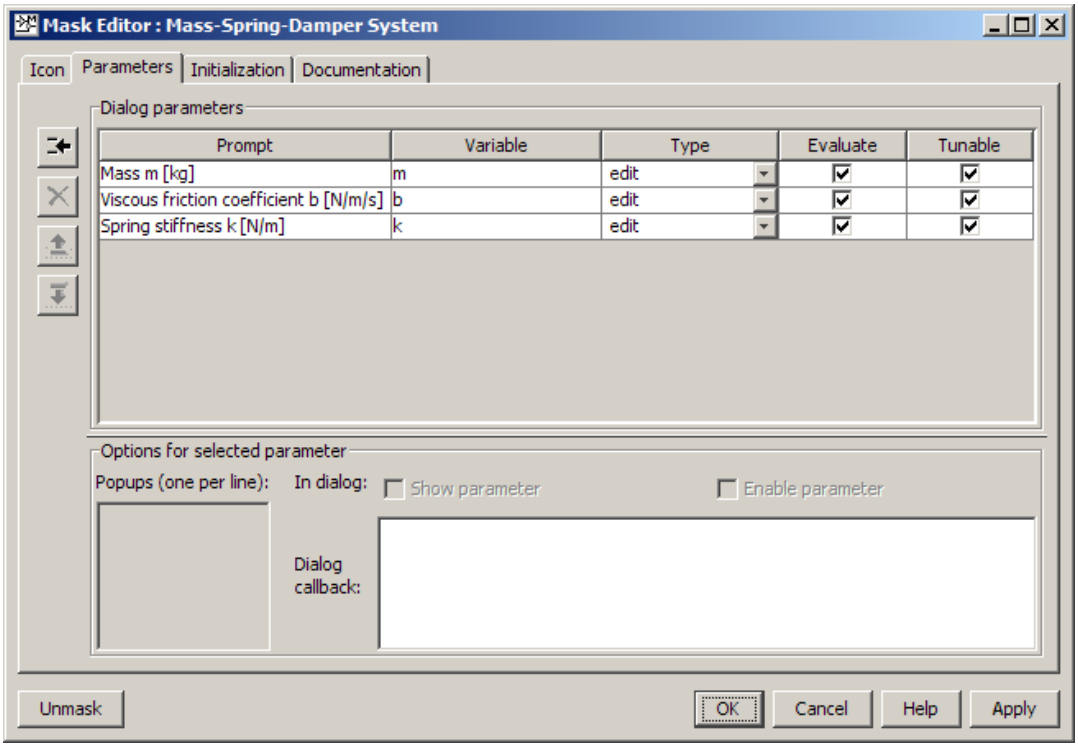

Figure 12 Parameters tab of the Mask Editor

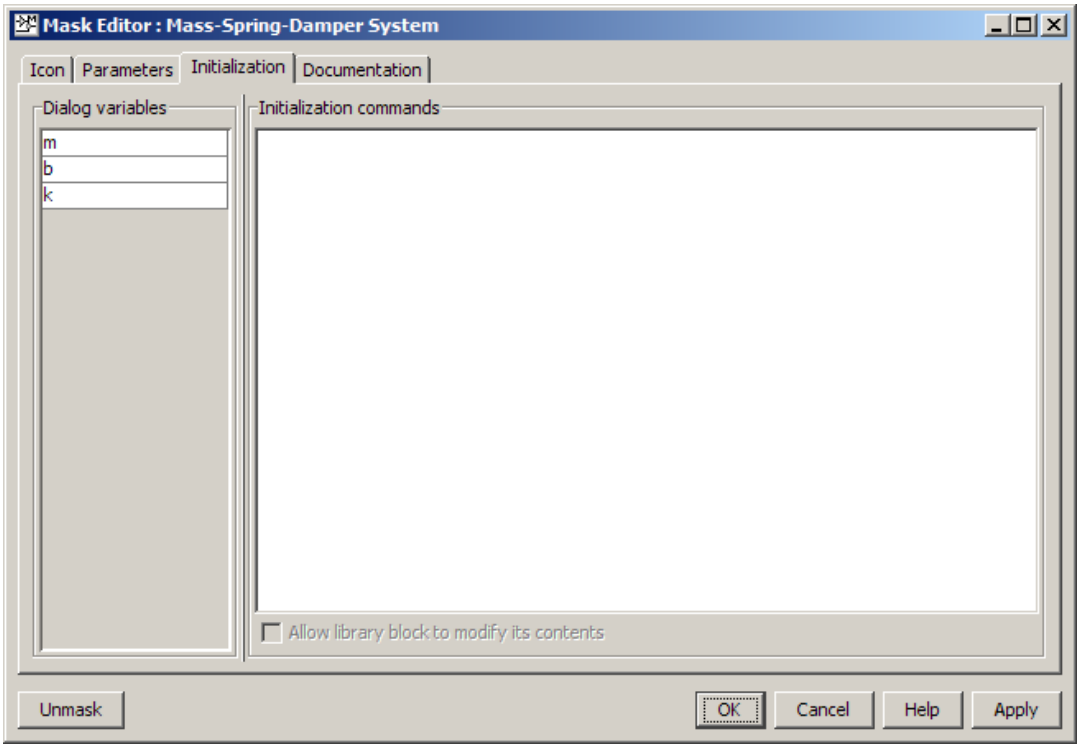

Figure 13 Initialization tab

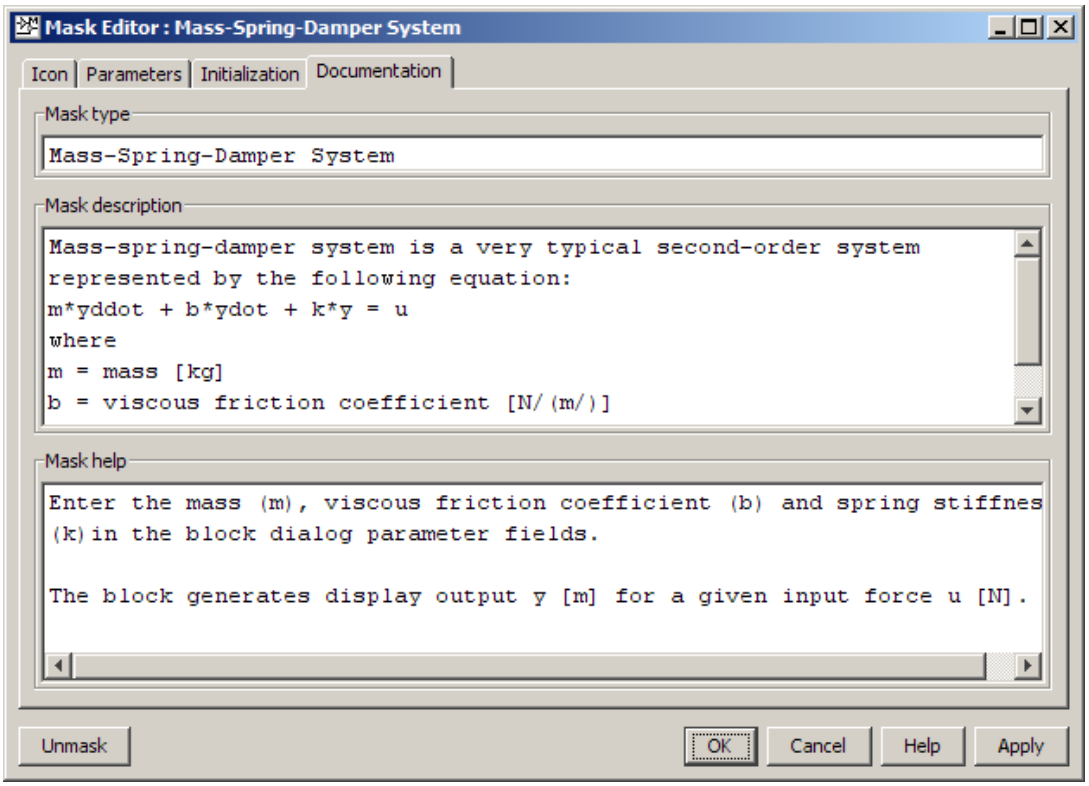

Figure 14 Documentation tab

- Click Apply button
- Click OK button. The resulting Simulink model looks like…

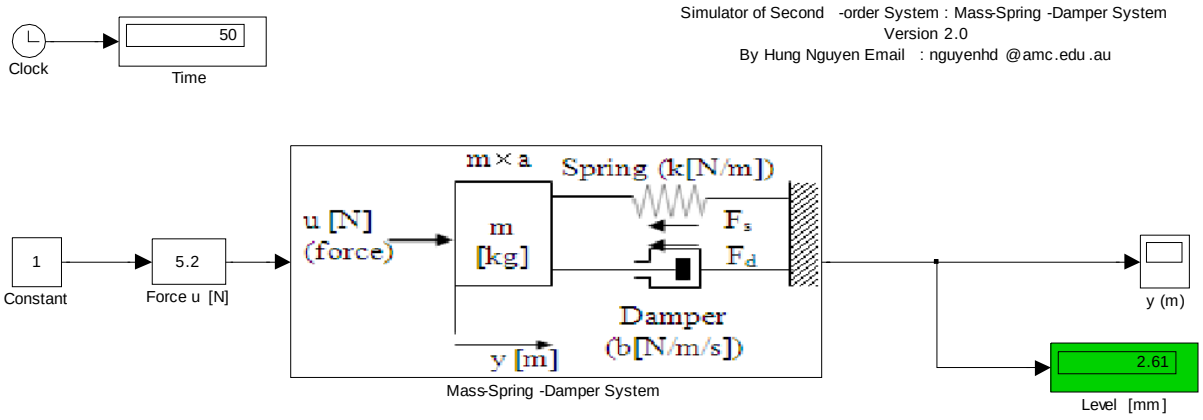

Figure 15 Simulink model with a masked block "Mass-Spring-Damper System"

- Double click the Mass-Spring-Damper System block.
- Enter values for m, b and k (as m = 20, b = 4 and k = 2) (you can try to click the Help button!)
- Click the Apply button, then the OK button.

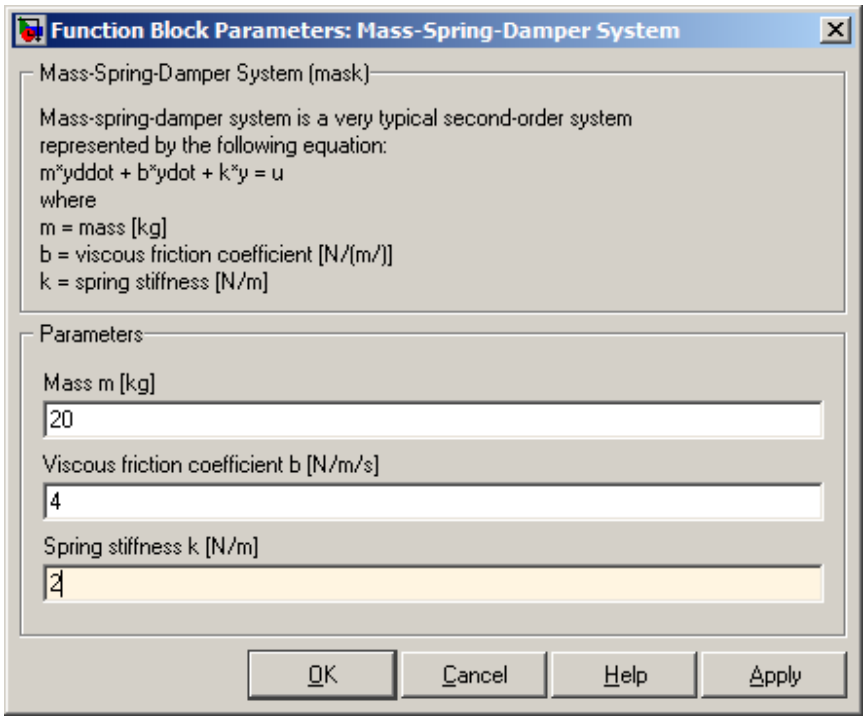

Figure 16 Mass-Spring-Damper System function block parameters

- Save the model.
- Run the model and test its functionality (you may need to clear all variables in Workspace). The same results should be obtained.

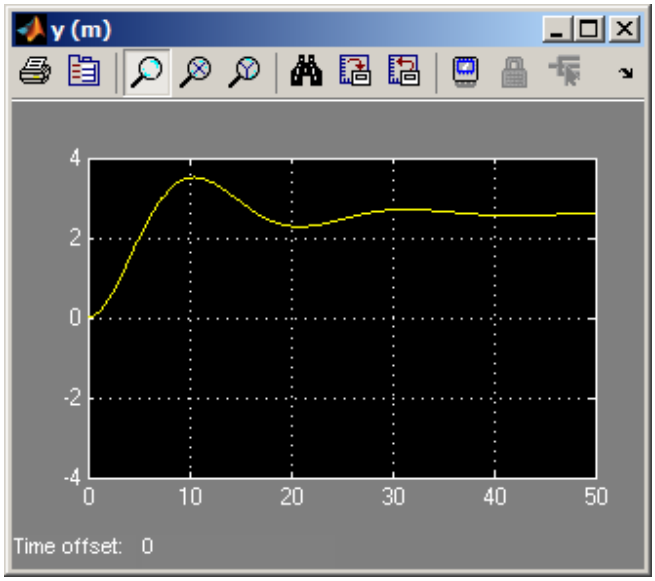

Figure 17 Response for an input force of 5.2 N

Note: You can compare results with http://techteach.no/simview/mass_spring_damper/index.php

Conclusions

At this point the following LOs have been met:

- Use Simulink to simulate dynamic systems (2nd-order systems)
- Create subsystem
- Mask subsystem

Follow-up Exercise

The following RLC network is represented by a second-order ODE below:

$$LC\ddot{e}_o + RC\dot{e}_o + e_o = e_i$$

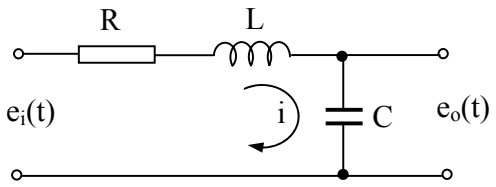

Figure 18 An RLC network

Make a Simulink model to simulate the network. Use these numerical values: R = 100 Ω, L = 0.2 H and C = 2000 μF, $e_i(t)$ = 5 V.