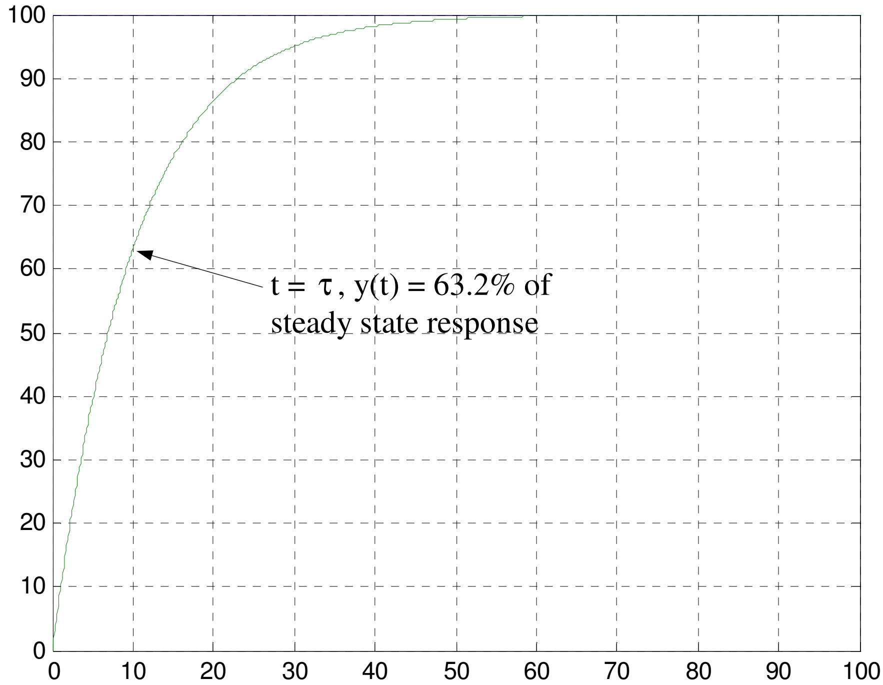
Appendix 2 Determination of the time constant from a trace produced by obtained data

A2.1 Response Method

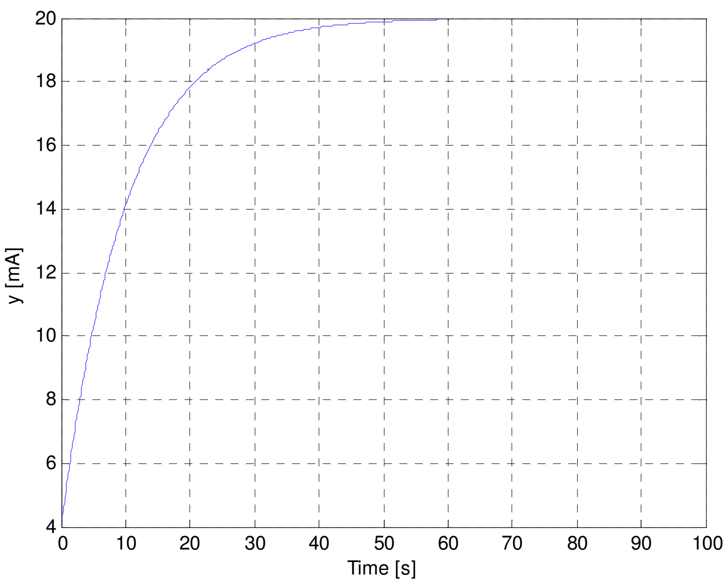
A first-order system is expressed by a first-order different equation below:

$$\tau \dot{y} + y = Ku$$

where τ is time constant (sec) and K is sensitivity. For example, τ = 10 seconds, K = 1 and the input is a step function u(t) = 100 (u(0) = 0), the solution is given below:



After an experiment has been carried out using the obtained data for some variables it is often required to find the underlying time constant with which the variable is changing. The obtained data consists of the system response vs time. Therefore if we can extract data and have a response as in the following figure we can find the time constant and sensitivity:



A2.2 Least Squares Algorithm Method

After experiments, if the following conditions are true:

a) the system is of first order
b) the disturbance causing the change in the variable is a step input

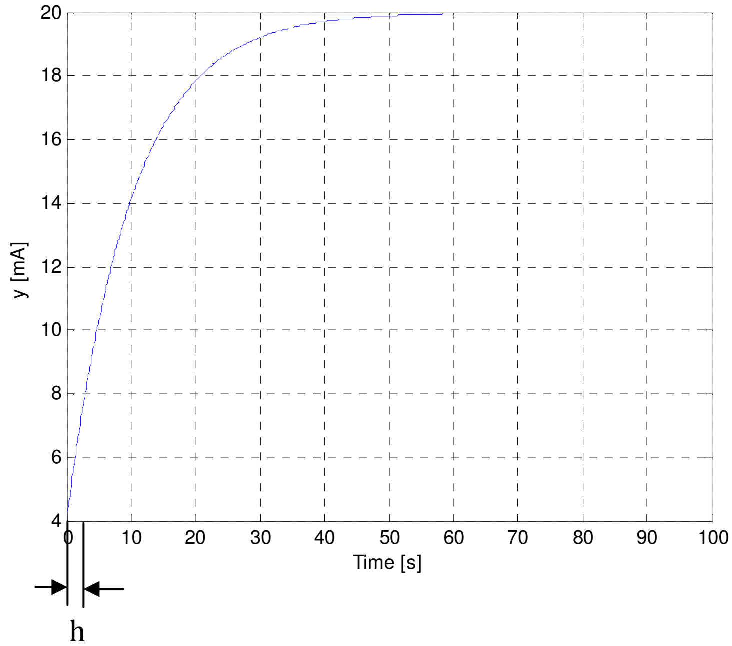
then the variable should be following one of the two possible functions; either

$y(t) = A + B\left(1 - e^{-\frac{t}{\tau}}\right)$ if the variable is increasing, or

$y(t) = C + De^{-\frac{t}{\tau}}$ if the variable is decreasing.

The major problems are that the values of A and C (the "d.c." component) and the values of B and D (the range) are unknown, also the time at which t = 0 is also unknown.

If the sampling interval (h) is known then a number of equally spaced (in time) values of the variable can be taken from the obtained data; assume that there are n+1 of these values y(0), y(1), y(2), … y(n+1). The first value occurs at some unknown time (call this time t_0) and can be determined by an extracted data from the obtained data.



The value of each of these values of the variable (assuming an increasing signal) must be given by:

$$y(0) = A + B\left(1 - e^{-\frac{T_0}{\tau}}\right)$$

$$y(1) = A + B\left(1 - e^{-\frac{T_0 + T}{\tau}}\right)$$

$$y(2) = A + B\left(1 - e^{-\frac{T_0+2T}{\tau}}\right)$$

$$\vdots$$

$$y(n) = A + B\left(1 - e^{-\frac{T_0+nT}{\tau}}\right)$$

If the difference between each of these values is taken, i.e. y(1)-y(0), y(2)-y(1),…, etc., then all the A values will disappear. The general case is given by:

$$\Delta y = y(k+1) - y(k) = \left[A + B\left(1 - e^{-\frac{T_0+(k+1)T}{\tau}}\right)\right] - \left[A + B\left(1 - e^{-\frac{T_0+kT}{\tau}}\right)\right]$$

$$= B\left(e^{-\frac{T_0+kT}{\tau}} - e^{-\frac{T_0+(k+1)T}{\tau}}\right) = Be^{-\frac{T_0}{\tau}}\left(e^{-\frac{kT}{\tau}} - e^{-\frac{(k+1)T}{\tau}}\right) = Be^{-\frac{T_0}{\tau}}e^{-\frac{kT}{\tau}}\left(1 - e^{-\frac{T}{\tau}}\right)$$

Now if the natural logarithm of the difference is taken the general case gives:

$$\ln(\Delta y) = \ln(B) + \ln\left(e^{-\frac{T_0}{\tau}}\right) + \ln\left(e^{-\frac{kT}{\tau}}\right) + \ln\left(1 - e^{-\frac{T}{\tau}}\right)$$

$$\ln(\Delta y) = \ln(B) - \frac{T_0}{\tau} - \frac{kT}{\tau} + \ln\left(1 - e^{-\frac{T}{\tau}}\right)$$

Note that each of these terms is a constant except the third term, therefore a graph of the natural logarithm of the differences against time will be a straight line with a negative slope. The reciprocal of the slope will be the time constant τ in seconds.

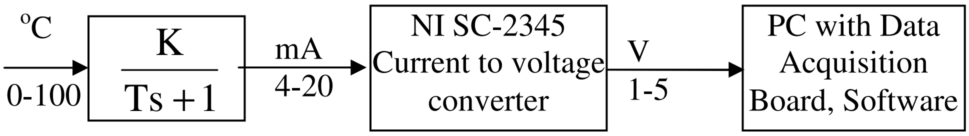
Appendix 3 Hints for Determination of Time Constant Using Data File

1. The MAT-formatted data file can be loaded into Workspace of MATLAB by the following command:
   >>load (the name of data file without extension)

2. Using the “plot” function you can do plotting of graphs (current/temperature versus time):
   >>plot(data(:,1),data(:,2));grid

3. To extract a matrix (e.g. data) use the following command
   >>time = data(:,1) % to extract column 1 of matrix “data”
   >>current = data(:,2) % to extract column 2 of matrix “data”
   >>row1 = data(1,:) % to extract row 1 of matrix “data”.

4. Use the following diagram for the computer-based temperature measurement system:



Thermocouple Temp Transmitter

From the obtained data the relationship between the input and output of the temperature transmitter can be determined as:

- Relationship between output current (4-20mA) and temperature input (°C) or
- Relationship between output voltage (1-5 V) and temperature input (°C)

5. To write data to an Excel file refer to the following codes:

```
% Compute values of the function y = exp(x), x = 0 to 1:
     x = 0:0.1:1; y = [x; exp(x)];
     success = xlswrite('MyExcelTest01',y');
 % End of Program
```

6. To read data from an Excel file refer to the following codes:

```
% Read data from an Excel file using xlsread:

   clear
   A = xlsread('MyExcelTest01');
% End of Program
```

7. To write and read data from/to a text-formatted file, refer to the following codes:

7.1 Writing a file: The following statements:

```
% Writing a file:
   x = 0:0.1:1; y = [x; exp(x)];
   fid = fopen('datafile.dat','w');
   fprintf(fid,'%6.2f   %12.8f\n',y);
   fclose(fid);
```

create a text file containing a short table of the exponential function as follows:

```
0.00   1.00000000
0.10   1.10517092
 …
```

```
        1.00   2.71828183
```

If you run a program with the above statements, a text-formatted data file should be created in the current directory.

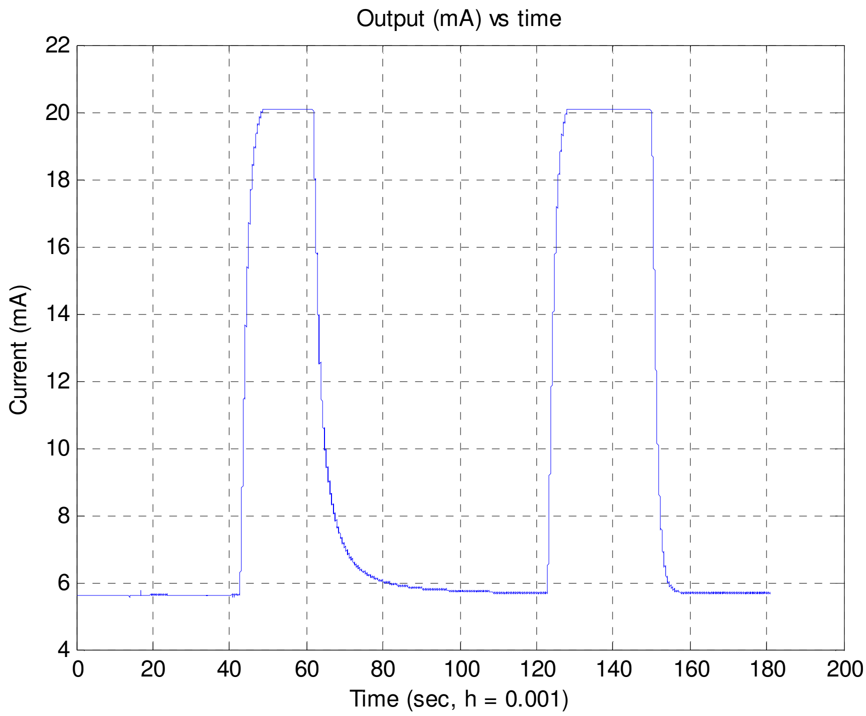
7.2 Reading a file: In order to read data from a text formatted file, we need to use fopen, fgets and fscanf. Look at the following codes to read data from the text file (datafile.dat) created in Example 7.1:

```
% Tute10Example02.m
% This program reads data from a text formatted file:

clear

fid = fopen('datafile.dat','r');
[A,count] = fscanf(fid,'%f %f\n',[2,11]); % [col,row]

fclose(fid);
```

Running this program, we obtain the data below:

```
>> A'
ans =
     0          1.0000
     0.1000     1.1052
     0.2000     1.2214
     0.3000     1.3499
     0.4000     1.4918
     0.5000     1.6487
     0.6000     1.8221
     0.7000     2.0138
     0.8000     2.2255
     0.9000     2.4596
     1.0000     2.7183
```

8. To extract data from a long vector, refer to the following example:
Example: run01.mat is a MAT-formatted data file obtained form an experiment containing a matrix with two columns (time and output current):



The sample interval is 0.001 and the length is 180801. You can extract data from 42.5 seconds to 50 seconds by the following codes:

```
clear
load run01
t=data(:,1);
y=data(:,2);
t2=t(42501:50000);y2=y(42501:50000);plot(t2,y2);grid
```