A Numerical Study of the Resistance of Transom-Stern Monohulls

Lawrence J. Doctors
The University of New South Wales, Sydney, NSW 2052, Australia
L.Doctors@UNSW.edu.au Volker Bertram

Abstract

Previous experimental data for the wave profiles behind a series of transom-stern ship models has been re-analyzed in this paper. Convenient and accurate regression formulas for the ventilation of the stern and the length of the transom hollow are provided here.

Predictions of the wave resistance and, hence, the total resistance can now be improved by using this approach, together with the traditional thin-ship formulation for resistance. However, the accuracy of these predictions is improved further if one also considers that there must be a minimal effective hollow length based on the flow behind the well-known backward-facing step.

These theories, together with the use of form factors, are applied to a series of fourteen high-speed monohull vessels. It is shown that these methods provide a high degree of predictive accuracy.

1. Introduction

1.1 Previous Work

A large proportion of the high-speed vessels currently being constructed or being used in passenger service share the characteristic of possessing a cut-off, or transom, stern. This feature of the vessel defies simple hydrodynamic analysis because the extent and detailed shape of the hollow cavity in the free surface created behind the vessel are unknown. They must be part of the mathematical solution to the problem. In principle, it is known that the pressure acting on the surface of the hollow must be atmospheric, and that the flow must separate from the transom tangentially. A proper analysis of the problem would require a fully three-dimensional treatment in which it would be necessary to iterate the geometric shape of this hollow, until the relevant kinematic and dynamic conditions were satisfied.

This is no easy task, given that some aspects of the geometry are highly nonlinear (and perhaps even intractable) within the framework of potential-flow theory. In particular, the closure of the hollow, which seems always to be accompanied by a considerable amount of spray and unsteadiness in the flow, appears to be a formidable problem in fluid mechanics. This region of the flow is usually referred to as a rooster tail because of the shape of the spray thrown into the air.

A small number of researchers has addressed this question in an attempt to gain some understanding of the flow. An early paper was written by Milgram (1969), in which the wake behind a vessel was modelled as a region filled with deadwater. The flow was assumed to separate in various ways near the stern and, in the more refined version of his approach, the separation was taken to occur tangentially to the hull surface. The traditional thin-ship theory of Michell (1898) was applied to the resulting hypothetical “body” encompassing both the hull itself and the wake region. A similar approach was taken by Doctors and Day (1997) for the prediction of the resistance of transom-stern vessels at high speeds, in which it was assumed that the hollow extended to infinity, and a hydrostatic correction was employed to the total resistance.

Another study, published by Tulin and Hsu (1986), specifically addressed the stern flow and the appropriate method of applying the abovementioned condition of constant atmospheric pressure on the surface of the hollow. The comment was made that the traditional wave-resistance problem may
not be properly posed and that the resulting computed wave resistance may not be unique in value! In their theory, the Kutta condition was essentially applied at the girth of the transom stern. The dilemma of the analysis of the rooster tail was apparently side-stepped by limiting the theory to a very high Froude number $F = \frac{U}{\sqrt{gL}}$, in which $U$ is the speed of the vessel, $g$ is the acceleration due to gravity, and $L$ is the wetted length of the vessel when at rest. In this limiting case, the rooster tail would be located far behind the vessel. Good agreement with experimental measurements of resistance of Series 64 vessels was demonstrated.

Molland, Wellicome, and Couser (1994b) tested different types of sink models in order to close the water flow behind the vessel. In this approach, one must ensure that the total source strength, for both the vessel itself and the hollow region, should be zero. They felt that their results were not as promising as they had hoped. In particular, they wished to be able to predict the resistance at lower Froude numbers. Consequently, they next employed in their research what one might call an engineering model of the transom hollow. In this approach, an experimentally determined length-to-width ratio for the hollow was assumed and the hollow was then incorporated into the stern of the vessel. Further results on this and related research have been presented by Insel and Molland (1991), Molland, Wellicome, and Couser (1994a and 1995), and Couser (1996).

The empirical correction of linearized wave-resistance theory was studied in a series of collaborative papers by Doctors, Renilson, Parker, and Hornsby (1991), Doctors and Renilson (1992 and 1993), and Doctors (1995a and 1995b). In these studies, both catamarans and monohulls were tested in a towing tank in water of various depths. Attempts to correlate the experimental results for the resistance with the linearized theory were then made. It was found that the theory could be used quite accurately to predict the effects of changes in the water depth or the spacing between the demihulls of a catamaran. In addition, the influences of sloping river banks could also be included in an approximate manner. However, a different correction factor was required at each speed, limiting the utility of the approach to some extent.

Mention should be made here of the work of Hanhirova, Rintala, and Karppinen (1995). These authors also fitted a Michell-type formula to a set of experimental results. They performed a regression analysis on the difference between such a prediction and the measurements.

A more sophisticated approach to estimating the length of the transom-stern hollow was proposed by Doctors and Day (1997) using their “fire-hose” model. In this approach, the length of the hollow was computed by a technique which allowed it to increase in a realistic manner with the speed, unlike the fixed-length concept of Molland and his coworkers cited above.

However, there was no confirmation that the length of the hollow was indeed correct, despite the fact that the predictions of resistance were in very good agreement with model experimental data. The
transom was assumed to be fully ventilated at all speeds, which meant that the transom-stern drag, associated with the missing hydrostatic pressure on the stern, was too high at low speeds.

1.2 Current Work

Subsequent to this initial effort, Robards and Doctors (2003) presented the outcome of a very extensive set of tests on a series of five ship models possessing a rectangular transom, and ballasted to five different drafts. This led to 25 combinations of geometry. The transom ventilation and the centreplane profile of the wave elevation were both measured for a range of speeds. This permitted a more scientific estimate of the desired parameters | the degree of transom ventilation and the transom-hollow length. This data has been reanalyzed a number of times; by Doctors (2003), Doctors and Beck (2005), and Doctors (2006a and 2006b).

The last publication forms the basis of the current project. In that paper, the analysis of the transom-stern data yielded two regression formulas. The first formula is for the ventilated or “dry” proportion of the transom, which is expressed as follows:

\[
\eta_{dry} = \frac{F_T}{T} = C_1 F_T^{C_2} \left( \frac{B}{T} \right)^{C_3} R_{NT}^{C_4}
\]  

(1)

Here, \( F_T = \frac{U}{\sqrt{gT}} \) is the transom-draft Froude number, \( B/T \) is the beam-to-draft ratio, and \( R_{NT} = \sqrt{gT^3/\nu} \) is the Reynolds number. The other symbols are the transom draft \( T \) and the kinematic viscosity of the water \( \nu \).

A second regression formula, of the same type, was derived for the "apparent" dimensionless hollow length, as follows:
The supposed geometry of the flow is presented in Figure 1(a), in which the symbols are also indicated. The adjective “apparent” is emphasized above, because the formula is only suited to the shape of the measured free surface. Because of the presence of the stagnant water in the hollow at low speeds, it is reasonable to assume that a minimum effective wavemaking length should be based on including the entire stagnant-water region.
The derived coefficients in Equation (1) and Equation (2) are presented in Table 1 and Table 2, respectively. In both cases, two sets of coefficients are listed, based on two methods of analysis. The first method, referred to as Static, uses the at-rest value of the transom-stern draft in order to effect the calculations and the subsequent prediction of the resistance. The second method, referred to as Dynamic, employs the underway transom draft instead. It can be observed that the ventilation varies approximately with the square of the speed while the apparent hollow length varies approximately with the cube of the speed.

The purpose of the present research is to apply this approach to a practical series of ship hulls. In this way, we are continuing the work of Sahoo, Doctors, and Renilson (1999) for monohulls, Sahoo and Doctors (2003) for the effects of restricted depth, and Sahoo, Doctors, and Pretlove (2006) for a staggered-demihiull catamaran.

2. Experiments in Towing Tank

2.1 AMECRC High-Speed Monohulls

The models were tested in the towing tank at the Australian Maritime College (AMC) in Launceston, Tasmania. The principal dimensions of the towing tank are presented in Table 3. The depth of the water, namely 1.500 m, was sufficient for the depth Froude number $F_d = \frac{U}{\sqrt{gd}}$ to be less than unity over most of the speed range of interest.

<table>
<thead>
<tr>
<th>Table 3: Particulars of Towing Tank</th>
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<tr>
<td>Particular</td>
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<tr>
<td>Overall length</td>
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<tr>
<td>Width</td>
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<td>Water depth</td>
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<th>Table 4: AMECRC Systematic Series</th>
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<td>Model</td>
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The principal geometric characteristics of the fourteen ship models, constituting the AMECRC high-speed monohull series, appear in Table 4. The table lists the displacement $\Delta$, the length-to-beam ratio $L/B$, the beam-to-draft ratio $B/T$, the block coefficient $C_B$, the prismatic coefficient $C_P$, and the slenderness coefficient $L/V^{1/3}$. The models have been
described in some detail by Bojovic (1995a and 1995b) and Bojovic and Goetz (1996). The models all possess a waterline length \( L \) of 1.6 m. Six of the fourteen models are presented in the six parts of Figure 2.

These hulls were chosen as being representative of high-speed patrol vessels. However, it may be noted that some of the dimensions, such as the beam-to-length ratio are large for this application.

2.2 Series of Investigations

The fourteen model vessels were all tested in relatively deep water up to a length Froude number of unity. Additionally, some finite-water-depth tests were conducted. As reported by Bojovic (1996), some variations in the displacement \( \Delta \) and the longitudinal centre of gravity LCG were also studied.

The tests were conducted in the towing tank at the Australian Maritime College during the period 1994 to 1996.

3 Numerical Approach

3.1 Mesh Refinement

Figure 3 is employed here to illustrate the relevance of the appropriate computational mesh to be used in the evaluation of the wave-resistance component. As seen in Figure 1(b), the hull is gridded longitudinally with \( N_x \) panels and the hull is gridded vertically with \( N_z \) panels. For the six vessels presented here, it can be observed from the plots that a mesh of 60 x 20 is more than adequate to represent the hull accurately.

![Figure 3: Refinement of Mesh (a) Model 1](image1)

![Figure 3: Refinement of Mesh (b) Model 2](image2)

![Figure 3: Refinement of Mesh (c) Model 3](image3)

![Figure 3: Refinement of Mesh (d) Model 4](image4)
While there are some relatively large differences at low Froude numbers, when a coarser mesh is employed, it must be borne in mind that these are plots of the coefficient of wave resistance, defined by

\[ C_w = \frac{R_w}{\frac{1}{2} \rho U^2 S} \]

surface of the vessel. Thus, the absolute error in the wave resistance is seen to be unimportant at these low Froude numbers.

The theoretical curves in the plots of Figure 3 all display a step reduction in resistance at a Froude number of 0.9682. This point in the speed range corresponds to a depth Froude number of unity. This drop is related to the loss of the transverse-wave component at this speed. The phenomenon is less significant as the width of the channel is increased.

3.2 Resistance Components

The resistance components for the six chosen vessels are given in Figure 4. The total resistance \( R_T \), the wave resistance \( R_w \), the hydrostatic (or transom-stern) resistance \( R_H \), and the frictional resistance \( R_F \) are all rendered dimensionless by the vessel weight \( W \). The frictional resistance is computed using the ITTC 1957 correlation line described by Lewis (1988). The experimental data appears as a series of symbols.

It is most encouraging to see the excellent correlation between the predictions for the total resistance, which is effected as a simple sum of the components, with the experimental data. This is true, even for the relatively fat Model 5.
4. Theoretical Model for Transom-Stern Flow

We now turn to Figure 5, in which different theoretical models for the transom-stern flow are compared for accuracy. Table 5 should be consulted for the explanation of the two codes describing the theory for the transom-water-level drop and the transom-hollow length, respectively.

In all the cases depicted here, there is an improvement in predictive accuracy, if (a) one uses a fitted regression curve to model the water drop rather than assuming the transom is fully ventilated, (b) the regression is based on coefficients using the static transom dimensions rather than the dynamic transom dimensions, and (c) the backward-facing-step reattachment length is used to place a lower limit on the effective transom-hollow length.
In addition to these comments, the application of a wave-resistance form factor $f_W$ of 0.8928 and a frictional-resistance form factor of 1.192, in the equation for total resistance, namely:

$$R_T = f_W R_W + f_F R_F,$$

(3)

generally increases the accuracy of the prediction. These two factors were obtained through a root-mean-square regression analysis of all the 28 test conditions for the AMECRC series.

5. Variation of Test Conditions

5.1 Change in Displacement

As an additional challenge, we consider the effect of changing the displacement, with zero trim, of two of the vessels (Model 6 and Model 9), in the two parts of Figure 6. In these and the subsequent graphs, the difference in specific total resistance $\Delta R_T/W$ is plotted. The difference is calculated relative to the 100% load case at zero trim.

In a general sense, it can be argued that the methodology works to predict these changes in Figure 6(a). The estimates for the effect of load change are slightly overpredicted when the load is increased beyond 100%. Of course, it must be remembered that the main effect of change in displacement (either -10%, +10%, or +20%) is accounted for by the form of the chosen plotted parameter. That is, the method of plotting greatly exaggerates any errors. In truth, all of the predictions for the total resistance are accurate to within one or two percent in an absolute sense.
The predictions for these small relative changes is slightly worse for Model 9 in Figure 6(b).

Table 5: Theories for the Transom-Stern Hollow

<table>
<thead>
<tr>
<th>Code</th>
<th>Method</th>
<th>Code</th>
<th>Method</th>
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<tbody>
<tr>
<td>Full</td>
<td>Transom is fully ventilated</td>
<td>Fit,D</td>
<td>Fitted regression curve using coefficients based on dynamic draft</td>
</tr>
<tr>
<td>Fit,D</td>
<td>Fitted regression curve using coefficients based on dynamic draft</td>
<td>Fit,S</td>
<td>Fitted regression curve using coefficients based on static draft</td>
</tr>
<tr>
<td>Fit,S</td>
<td>Fitted regression curve using coefficients based on static draft</td>
<td>Fit,S,B</td>
<td>As for Fit,S but assuming a minimal value based on flow past a backward-facing step</td>
</tr>
</tbody>
</table>

5.2 Change in Trim

Finally, the influence of change in the bow-down trim angle $\beta$ is depicted in the two parts of Figure 7, for the same two models. This exercise poses an even greater challenge than the previous exercise in which the displacement was changed.

Figure 6: Change in Displacement (a) Model 6

Figure 6: Change in Displacement (b) Model 9

Figure 7: Change in Displacement (a) Model 6

Figure 7: Change in Displacement (b) Model 9
Nevertheless, the numerical values of the predictions are more than acceptable for a Froude number less than about 0.35. Also, in a general sense, the ordering of the predictions matches that of the experimental data at high values of the Froude number.

6. Conclusions

This research has demonstrated that the enhanced models for the water flow behind the transom lead to much more accurate estimates of the transom hydrostatic drag and the wave resistance. As a consequence, the total resistance is predicted with much better accuracy. In particular, it can be noted that the correlation of theory and experiment is now exceptionally good for the difficult low-speed region of the resistance curve.

A start to determining suitable form factors for the resistance components demonstrates that even further improvements to the accuracy of the theory are achievable. It is seen that it is advisable to choose a wave-resistance form factor less than unity and a frictional-resistance form factor greater than unity. These statements are in accord with previous, preliminary, work on this matter. The need for a frictional-resistance form factor greater than unity has also been reported by other researchers.

It is recommended that further effort be invested in clarifying the proper values for the two form factors. It is suggested that these factors should depend on the actual vessel geometry.

7. Acknowledgments

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8. References


