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Production of this document is based solely on the objective of providing the students a comprehensive set of notes on Ship Dynamics (most of it is based on the book “Dynamics of Marine Vehicles” written by Prof. R Bhattacharyya). Hence as far as possible due care has been taken to reproduce material from various sources such as books, journals, conference papers etc with due acknowledgement to various authors and sources. It is not the intention of the author to ignore, delete or misrepresent information in this book. If there is any missing information then this has taken place inadvertently and expected to be corrected in future editions.

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CONTACT INFORMATION

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Unit Specification

Unit Code: E03 305  Unit Title: Ship Dynamics
Pre-requisite: E03 281  Fluid Mechanics
            E03 712  Calculus of Several Variables

Semester: 2  Year: 2006
Course: Bachelor of Engineering
Dept/Sch: Maritime Engineering
Campus: Newnham

Fraction of an EFTSL: 0.125

Academic Staff: Dr. Prasanta Sahoo & Mr. Jonathan Duffy

<table>
<thead>
<tr>
<th>Title</th>
<th>Name</th>
<th>Office</th>
<th>Email</th>
<th>Phone</th>
<th>Consultation days &amp; times</th>
</tr>
</thead>
<tbody>
<tr>
<td>Co-ordinator</td>
<td>Prasanta Sahoo</td>
<td>F-84</td>
<td><a href="mailto:P.Sahoo@amc.edu.au">P.Sahoo@amc.edu.au</a></td>
<td>63354822</td>
<td>Wednesdays 0900-1100 hrs</td>
</tr>
</tbody>
</table>

AIM OF THE UNIT:

- To introduce the concepts of ship behaviour in waves.
- To understand the process of evaluating the seakeeping characteristics of a vessel.

LEARNING OUTCOMES:

On successful completion of this unit, students should be able to:

- Explain the concepts of simple harmonic motion as applied to ship seakeeping and estimate, the natural frequency for any floating object in roll/pitch/heave, including the effects of damping.
- Explain the principles and limitations of linear strip theory and be able to calculate the forces and moments on a vessel given the 2-D added mass and damping coefficients of the strips.
- Calculate two-dimensional added mass coefficients from conformal mapping techniques, including Lewis forms.
- Generate wave spectra from finite wave height records in the time domain.
- Explain the principles of motion in irregular waves, including wave spectrum and motion spectrum, and calculate the effect of forward speed on encounter spectrum.
- Explain the principles behind, and conduct, simple model experiments to predict the seakeeping of a vessel.
- Explain the influence of a seaway on the power required to propel a vessel including both voluntary and involuntary speed reduction.
- Explain MSI from the seakeeping characteristics of a vessel.
• Explain the concepts involved with the conventional equations of motion governing ship manoeuvring, including the physical meaning of the coefficients, non-linearities and non-dimensionalisation.

• Explain the principles of directional stability, and be able to demonstrate whether a ship is directionally stable from knowledge of the “stability derivatives”.

• Explain the effect of the rudder and propeller on manoeuvring of a ship both at low speed and at normal cruising speed.

• Describe the different types of model experiments used to predict ship manoeuvring behaviour and explain their rational and limitations.

• Explain the forces acting in the different phases of a turn and their influence on turning behaviour of a ship, including heeling.

• Explain slender body theory as it applies to the prediction of the manoeuvring coefficients.

• Explain the principles and conduct of manoeuvring trials.

• Explain the effect of restricted water on the manoeuvring characteristics of a vessel.

Primary Delivery Mode:

On Campus Video Distance WWW/email

Web-CT: Yes/No

Teaching and Learning Approaches:

Lectures, small group assignments, team project work and team based experimental investigation.

Textbook:

None.

Reference Texts:


Bhattacharyya, R., Dynamics of Marine Vehicles, John Wiley & Sons, New York, 1982


Required Materials:

Prescribed Calculators are non-programmable, preferably Casio-FX 82 model or equivalent. For experimental work in towing tank and/or field trips students are required to satisfy the safety requirements as prescribed.
**Extra Costs:**

Lecture notes available through Student Association may be subject to a small levy. It is also available on the web located at [http://academic.amc.edu.au/~psahoo/Subjects](http://academic.amc.edu.au/~psahoo/Subjects).

**Materials to be provided by AMC:**

None. Students do have access to AMC facilities such as CAD Labs where appropriate hardware and software exist on a 24/7 period.

**Health and Safety requirements:**

Where appropriate, e.g. with regard to field trips, laboratory work, work aboard AMC vessels, etc, the health and safety requirements are to be strictly adhered to.

**Class times:**

<table>
<thead>
<tr>
<th>Class</th>
<th>Day</th>
<th>Time</th>
<th>Locations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lecture</td>
<td>Monday</td>
<td>1310-1500</td>
<td>G-71/72</td>
</tr>
<tr>
<td></td>
<td>Tuesday</td>
<td>0900-1100</td>
<td>G-71/72</td>
</tr>
<tr>
<td>SHC</td>
<td>TBA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Field Trip</td>
<td>None</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consultation</td>
<td>Wednesday</td>
<td>1000-1100</td>
<td>F-84</td>
</tr>
</tbody>
</table>

**Attendance Requirements:**

Attendance at all assigned class times is expected. You are responsible for all information (both academic and administrative) presented during class times. Should you miss a class for whatever reason it is your responsibility to obtain information and content that was missed.

**Syllabus and Learning Schedule:**

1. Seakeeping (34 Hours):
   a) **Motion Equations**
      Application of simple harmonic motion, natural frequency, and effect of damping.
   b) **Force Calculation**
      Calculation of exciting forces, added virtual mass, conformal mapping, and Lewis forms.
   c) **Motion in the Frequency Domain**
      Encounter frequency, wave spectrum, and motion in an irregular seaway.
   d) **Vessel Motions**
      Dynamic stability and capsizing, powering in a seaway, extreme motions, and human factors.
   e) **Motion Prediction of High-Speed Craft**
      Seakeeping characteristics of high-speed monohull and multihull vessels and its relevance to seakeeping index and MSI.

2. Manoeuvring (18 Hours):
   a) **Equations of Motion**
      The equations of motion; directional stability, non-linearities, and prediction of the coefficients.
   b) **Factors Influencing Turning**
      The turning path of the ship, the effect of hull shape and the propeller, and the effect of restricted water.
c) Trials and Experiments
Model experiments; manoeuvring trials, and IMO regulations.

<table>
<thead>
<tr>
<th>Week</th>
<th>Date</th>
<th>Topic</th>
<th>Readings/Problems</th>
<th>Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10 July 2006</td>
<td>Intro. to unit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>11 July</td>
<td>Solutions to Vibration Equations</td>
<td>Chapter 1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>17/18 July</td>
<td>Heaving motion</td>
<td>Chapter 2</td>
<td>Problem solving</td>
</tr>
<tr>
<td>4</td>
<td>24/25 July</td>
<td>Added mass, damping and exciting force</td>
<td>Chapter 2</td>
<td>Problem solving</td>
</tr>
<tr>
<td>5</td>
<td>31 July &amp; 01 Aug</td>
<td>Pitching Motion</td>
<td>Chapter 3</td>
<td>Problem solving</td>
</tr>
<tr>
<td>6</td>
<td>7/8 Aug</td>
<td>Rolling motion</td>
<td>Chapter 4</td>
<td>Problem solving</td>
</tr>
<tr>
<td>7</td>
<td>14 Aug</td>
<td>Radius of gyration</td>
<td>Chapter 5</td>
<td>Problem solving</td>
</tr>
<tr>
<td>8</td>
<td>15 August</td>
<td>Class Test on Seakeeping</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>21/22 Aug</td>
<td>Waves and properties</td>
<td>Chapter 6</td>
<td>Problem solving</td>
</tr>
<tr>
<td>10</td>
<td>28 Aug to 01 Sept</td>
<td>Non-teaching week</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>4/5 Sept</td>
<td>Irregular waves and wave spectrum</td>
<td>Chapter 7</td>
<td>Problem solving</td>
</tr>
<tr>
<td>12</td>
<td>11/12 Sept</td>
<td>Predicting Motion and added resistance</td>
<td>Chapter 8</td>
<td>Problem solving</td>
</tr>
<tr>
<td>13</td>
<td>17/18 Sept</td>
<td>Rolling motion</td>
<td>Chapter 4</td>
<td>Problem solving</td>
</tr>
<tr>
<td>14</td>
<td>25/26 Sept</td>
<td>Motion Stability/Control Forces</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>2/3 Oct</td>
<td>Model tests/Hydrodynamic Coeffs.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>9 Oct</td>
<td>Stability &amp; Control and Rudder Design</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>16/17 Oct</td>
<td>Class Test on Manoeuvring</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Indicates that group lab report is due within two weeks of completion of lab and to be submitted by 5 pm on Friday of that designated week.

Assessment:

<table>
<thead>
<tr>
<th>Coursework</th>
<th>Unit Mark %</th>
<th>Week/Date</th>
<th>Examination</th>
<th>Unit Mark %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assignment 1</td>
<td>30</td>
<td>11 22 Sept</td>
<td>Examination</td>
<td>100</td>
</tr>
<tr>
<td>Laboratories</td>
<td>20</td>
<td>TBA 2 weeks after date of lab</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Test 1 (Seakeeping)</td>
<td>25</td>
<td>6 15 Aug</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Test 2 (Manoeuvring)</td>
<td>25</td>
<td>14 10 Oct</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coursework Total:</td>
<td>100%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coursework</td>
<td>30%</td>
<td>Examination Total:</td>
<td>70%</td>
<td></td>
</tr>
<tr>
<td>Combined Total:</td>
<td></td>
<td>Combined Total:</td>
<td>100%</td>
<td></td>
</tr>
</tbody>
</table>

To pass this unit, a student must achieve a minimum of 50% in both course work and examination. In addition the combined average of the course work and exam marks must be at least 50%. The combined average for this subject will be calculated using a weighting ratio of 30% for course work and 70% for examination. Non-attendance or non-submission of labs/lab reports or assignments will result in a grade as "FAIL" for entire subject.

Length of final examination  = 3 hours
Length of Class Test      = 90-110 minutes

Notes:
The week/date column indicates the semester week number and the date that an assignment is due for submission. If you require exemptions from previous course work (repeating unit) then it must be clarified within the first week of the semester.

**Submission of assignments:**

Assignments will be submitted to the Division, Academic & Research Office (1st floor Swanson Building). Late submissions will receive up to a 5% penalty for each day of delay. Electronic submissions are acceptable provided the time stamp of email does indicate that it is submitted on time. Generally submission is due on the week ending on Friday at 1700 hours. The assessed work will be returned in lecture.

**Final grade:**

The grade that you receive for this unit will be determined by a committee of examiners. The raw marks that you receive from each piece of assessable material will be combined in order to determine a letter grade for the unit. The raw marks may undergo a scaling process to ensure meeting AMC policies on the distribution of grades.

**Problems with your assessment:**

If you have questions or problems with your assessment, you should undertake discussion with the following people until you have received a resolution of the issues. (1) The person who marked the assessment. (2) Unit Coordinator. (3) Head of Department/School in charge of the unit. (4) Vice-President (Academic & Research) – Professor Tom Hardy. If this does resolve the issue you may file formal appeal by contacting the office of the Registrar.

**Student Support:**

Some students will have problems that will affect learning that can span a wide range of family, relationship, health, emotional, financial and educational issues. AMC has support systems, but it is important that you recognise that you have a problem and seek help promptly before your learning is irreparably hampered. The student support offices at Newnham campus are located above the library. For a description of the support available at AMC please see [http://www.amc.edu.au/students/student.support/](http://www.amc.edu.au/students/student.support/).
Assignment Exercise

Introduction

The aim of this study is:

To provide students with knowledge and understanding of dynamics of a vessel in forward speed addressing the areas of motions in irregular seaway. In addition to this students would be able to understand the motion characteristics of different types of vessels.

Assignment Study

Each individual student is required to attempt one of the following topics:

- Write a report on seakeeping characteristics of monohull or multihull (catamarans, trimarans, swath etc.). OR
- Create a series of hull forms (NPL hull form will be given) and predict the motion characteristics and develop a simple spreadsheet based s/w to predict the response, when modelled as a monohull or catamaran.

Date due

Individual report to be submitted no later than 22\textsuperscript{nd} September 2006.

Assessment & Report

As per the assessment schedule this exercise carries only 30\% of the course work mark.

The assignments report should address the following points:

- Abstract of the report (min 200 words).
- Review of similar articles to provide background work done so far (min 1200 words, 4-5 pages)
- What various authors have achieved?
- Summary conclusions in your own words about various papers.
- The report should include all relevant figures, tables and equations.
- Finally the report should have all data necessary to calculate the dynamics if a person so desires.
- Conclusion and Final Remarks.
- Nomenclature and References (minimum 8 to 10)
- Follow the report format to write the report not exceeding 12 pages in total.
- Present your report using the report format located at http://academic.amc.edu.au/~psahoo/Subjects
- The report can also be emailed to me at P.Sahoo@ame.edu.au by due date.

General assessment is based on

- Presentation quality of report (20\%)
- Relevant theory, quality & quantity (may also include development of a spread sheet based s/w). (50\%)
- Discussion of work/results/findings (20\%)
- Conclusions (10\%)

Learning Outcomes

This investigative study is designed to address the following learning outcome, (Note: all of the following outcomes are not addressed in every assignment exercise):

- Explain the principles and limitations of linear strip theory and be able to calculate the forces and moments on a vessel given the 2-D added mass and damping coefficients of the strips.
Calculate two-dimensional added mass coefficients from conformal mapping techniques, including Lewis forms.
Generate wave spectra from finite wave height records in the time domain.
Explain the principles of motion in irregular waves, including wave spectrum and motion spectrum, and calculate the effect of forward speed on encounter spectrum.
Explain the principles behind, and conduct, simple model experiments to predict the seakeeping of a vessel.
Explain MSI from the seakeeping characteristics of a vessel.

Generic Attributes

- ability to apply knowledge of basic science and engineering fundamentals;
- ability to communicate effectively, not only with engineers but also with the community at large;
- in–depth technical competence in at least one engineering discipline;
- ability to understand problem identification, formulation and solution;
- ability to utilise a systems approach to design and operational performance;
- ability to function effectively as an individual and in multi-disciplinary and multi-cultural teams, with the capacity to be a leader or manager as well as an effective team member;
- understanding of the social, cultural, global and environmental responsibilities of the professional engineer, and the need for sustainable development;
- understanding of the principles of sustainable development;
- expectation of the need to undertake lifelong learning and the capacity to do so.
Laboratory Exercise

Introduction

The aim of this study is:

To provide students with knowledge and understanding of dynamics of a vessel in forward speed addressing the areas of heave and pitch motions in irregular seaway. In addition to this students would be able to extrapolate the model scale results of seakeeping and added resistance to a full-scale vessel based on the theory.

Thus, in addition to classroom lectures, tutorials, and assessments, the unit also includes a standard experiments to be conducted in the towing tank (AMC Ship Hydrodynamic Centre) designed to introduce the student to the principles involved model and full scale measurements. In order to successfully complete this exercise, the student must participate in a group and submit a laboratory report (group report). Note: the experimental results and the relevant theory will assist the student in completing the formal assessments.

Laboratory Experiments

Each group of students is required to complete all experiment as defined below:

- Seakeeping tests of a scaled model (heave and pitch) as the experimental guidelines of AMCSHC staff.
- Added resistance measurements of a scaled model as per experimental guidelines of AMCSHC staff.

The group list with dates and timings are located at [http://academic.amc.edu.au/~psahoo/Subjects/Seakeeping/](http://academic.amc.edu.au/~psahoo/Subjects/Seakeeping/)

Date due

A single group report to be submitted two weeks from the date of completion of the experiment or latest by 22nd September 2006.

Procedure

1. Labs should be carried out with due regard to OH&S requirements. Please consult in advance the AMCSHC staff regarding OH&S policy and any special requirements regarding dress code.
2. On completing each lab, the students must ensure that all equipment and material are replaced in the appropriate places, and the area left in a similar condition to that at the beginning of the lab. They must abide by the conditions imposed by AMCSHC staff during the course of the experiment.
3. Late lab reports will not be accepted.
4. Failure to submit or attend the laboratory exercise will result in a “FAIL” grade for the unit unless a medical condition exists on the day of experiment.

Assessment & Report

General assessment is based on

- Presentation quality of report (20%)
- Relevant theory, quality & quantity (may also include development of a spread sheet based s/w) (50%)
- Discussion of work/results/findings (20%)
- Conclusions (10%)

Labs must be written up as a formal group report, signed by all group members, and handed in within two weeks from the date of the lab. Needless to say that the lab report should follow the template located at [http://academic.amc.edu.au/~psahoo/Subjects/](http://academic.amc.edu.au/~psahoo/Subjects/) with the appropriate cover sheet.
General assessment is based on Report (10%), Numerical Results and Discussion (20%), Usefulness of Spread sheet (10%), Conclusions & Discussion (10%).

Learning Outcomes

This investigative study is designed to address the following learning outcome, (Note: all of the following outcomes are not addressed in every laboratory exercise):

- Conduct a practical resistance and powering estimation.
- Conduct and analyse a resistance experiment in the towing tank.
- Explain the principles of motion in irregular waves, including wave spectrum and motion spectrum, and calculate the effect of forward speed on encounter spectrum.
- Explain the principles behind, and conduct, simple model experiments to predict the seakeeping of a vessel.
- Explain the influence of a seaway on the power required to propel a vessel including both voluntary and involuntary speed reduction.

Generic Attributes

- ability to apply knowledge of basic science and engineering fundamentals;
- ability to communicate effectively, not only with engineers but also with the community at large;
- in–depth technical competence in at least one engineering discipline;
- ability to understand problem identification, formulation and solution;
- ability to utilise a systems approach to design and operational performance;
- ability to function effectively as an individual and in multi-disciplinary and multi-cultural teams, with the capacity to be a leader or manager as well as an effective team member;
- understanding of the social, cultural, global and environmental responsibilities of the professional engineer, and the need for sustainable development;
- understanding of the principles of sustainable development;
- expectation of the need to undertake lifelong learning and the capacity to do so.

Note: If required the laboratory sheets will be supplied by AMCSHC Staff (Manager: Mr. Gregor MacFarlane, ext: 4880).
Equation of motion in heave:

\[ a\ddot{z} + b\dot{z} + cz = F_o \cos \omega_e t \]

Solution to this equation

\[ z = e^{-vt} \left( C_1 \cos \omega_d t + C_2 \sin \omega_d t \right) + z_a \cos(\omega_e t - \varepsilon) \]

and when in calm water without damping

\[ z = A \sin \omega_z t + B \cos \omega_z t \]

or

\[ z = A \sin \left( \omega_z t - \varepsilon \right) \]

With damping

\[ z = Ae^{-vt} \sin(\omega_d t - \varepsilon) \]

\[ v = \frac{b}{2a} \]

\[ \omega_d = \sqrt{\omega_z^2 - v^2} \]

\[ w_z = \sqrt{\frac{c}{a}} \]

\[ k = \frac{v}{\omega_z} \]

\[ \mu_z = \frac{1}{\left( 1 - \Lambda^2 \right)^{\frac{3}{2}} + 4k^2 \Lambda^2} \]

\[ z_a = z_{stat} \mu_z \]

\[ \Lambda = \frac{\omega_e}{\omega_z} \]

\[ \varepsilon = \tan^{-1} \frac{2k\Lambda}{1 - \Lambda^2} \]
\[ z_{stat} = \frac{F_o}{c} \]

\[ c = \rho g LBC_{\varphi} \]

\[ k_{xx} = \sqrt{\frac{\sum w_i (y_i^2 + z_i^2)}{\sum w_i}} \]

\[ k_{yy} = \sqrt{\frac{\sum w_i (x_i^2 + z_i^2)}{\sum w_i}} \]

Equations for Pitch and Roll are exactly the same except the exciting force is replaced by exciting moment.

For pitch \( z \) is replaced by \( \theta \) and for roll \( z \) is replaced by \( \phi \).

\( c \) for pitch = \( \Delta GM_L \)

\( c \) for roll = \( \Delta GM_T \)

Properties of Waves

- Wave Length = \( L_w \)
- Wave Height = \( H_w \)
- Wave Period = \( T_w \)
- Wave Velocity = \( V_w \)
- (also celerity)
- Wave Frequency = \( \omega \)

\[ k = \frac{2\pi}{L_w} = \frac{\omega^2}{g} = \frac{g}{V_w^2} = \frac{4\pi^2}{gT_w^2} \]

Depth of water from free surface = \( z \)

Elevation of Lines of equal pressure

\[ \xi = \xi_0 e^{-kz} \cos k(x - V_w t) \]

Wave velocity in deep water

\[ V_w = \sqrt{\frac{gL_w}{2\pi}} \]

Wave velocity in shallow water

\[ V_w = \sqrt{gh} \quad (h = \text{depth of water}) \]

Pressure

\[ p = \rho g (z - \xi) \]

Wave slope

\[ \alpha_M = \frac{2\pi \xi_0}{L_w} \]

Energy per unit area

\[ \frac{1}{2} \rho g \xi_a^2 \]

Encounter frequency

\[ \omega_e = \omega_w \left( 1 - \frac{V}{V_w} \cos \mu \right) \]
\[
= \omega_w \left(1 - \frac{\omega_w V}{g} \cos \mu \right)
\]

where

\(\mu\) = heading angle

= 180° for head seas
= 0° for following seas
= 90° for beam seas

General expression for Rayleigh distribution is

\[
p(x_i > x) = e^{-x_i^2 / \bar{x}^2}
\]

\[
p(x_i) = \frac{2x_i}{\bar{x}^2} e^{-x_i^2 / \bar{x}^2}
\]

where \(p(x_i)\) is the probability density.

\[
\bar{x}^2 = \frac{\sum [x_i^2 \times f(x_i)]}{\sum f(x_i)}
\]

\(\bar{x}\) = RMS value


ITTC Spectrum:

\[
S(\omega_w) = \frac{A}{\omega_w^4} e^{-B/\omega_w^6}
\]

\(\omega_w\) = rad/sec

\(g\) = acc. due to gravity

\(A = 8.10 \times 10^{-3} \) g^2

\(B = 3.11 \times 10^{4}/H_{1/3}^2\) where \(H_{1/3}\) in cms.

\(H_{1/3} = 4 \sqrt{m_o}\)

\(H_{avg} = 2.50 \sqrt{m_o}\)

\(H_{1/10} = 5.1 \sqrt{m_o}\)

Energy = \(\rho g m_o\) per unit area.

\[
S(\omega_e) = S(\omega_w) \ast \frac{1}{\left[1 - \frac{4\omega_w V}{g} \cos \mu \right]^{1/2}}
\]

OR
\[ S(\omega_e) = \frac{1}{1-2\frac{\omega_n V}{g} \cos \mu} \]

RAO for heave = \( \left( \frac{z_a}{\xi a} \right)^2 \)

for roll = \( \left( \frac{r_a}{\xi a} \right)^2 \)

for pitch = \( \left( \frac{\theta_a}{\xi a} \right)^2 \)

Response

\[ S(z, \theta \text{ or } \phi) = S(\omega_e)^* \text{RAO} \]

\[ \frac{r_a}{\xi a} = \mu \phi \frac{\omega_n^2 \sin \mu}{g} \]

significant(response) = \( 4\sqrt{m_e} \)

Added Resistance in Waves

\[ \frac{R_{AW}}{\rho g L B^2} = \frac{L^2}{32B} \left[ \left( \frac{z_a}{\xi a} \right)^2 \right] + \left( \frac{L}{L_a} \right)^2 \left( \frac{\theta_a L}{W} \right)^2 \left( \frac{\xi a}{\theta_a} \right)^2 \left( \frac{z_a}{\xi a} \right) p_2 \cos \epsilon \]

\[ p_1 = \frac{2\omega_e^3}{g^2 L} \sqrt{\frac{g}{L}} \cdot B_{33} \]

\[ p_2 = \frac{2\omega_e^3 V}{g^2 L} \sqrt{\frac{g}{L}} (B_{35} + B_{53}) \]

\[ p_3 = \frac{8\omega_e^3 V}{g^2 L} \sqrt{\frac{g}{L}} \cdot B_{55} \]

\[ \epsilon = |\epsilon_z - \epsilon_\theta| \]

Heave displacement \( z = z_a \cos(\omega_e t) \)

Pitch displacement \( \theta = \theta_a \cos(w_e t + \epsilon) \)

\[ B_{33} = \frac{1}{\omega_e \sqrt{L g}} \int b_n d\xi \]
\[ B_{35} + B_{53} = \frac{2}{\omega_e \sqrt{\frac{g}{L}}} \int \xi b_n \, d\xi \]
\[ B_{55} = \frac{1}{\omega_e \sqrt{\frac{g}{L}}} \int \xi^2 b_n \, d\xi \]

\( \nabla = \) volumetric displacement \( b_n = \) sectional damping coefficient

\( \xi = \frac{2x}{L} = \) non dimensional longitudinal coordinate.